STIFFNESS OF STEPPED BARS

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Monterey, California

THESIS

STIFFNESS OF STEPPED BARS

by

Dennis Michael Doyle

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Thesis Advisor: R. E. Newton

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3. circular cross section under axial load;

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Stiffness of Stepped Bars

by

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Submitted in partial fulfillment of the
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4. circular cross section under torsional load.

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LIST OF SYMBOLS

A = cross-sectional area (in.^{2})

[B] = matrix relating the displacement vector to the strain vector

d = smaller diameter for bar with circular cross section (in.), or smaller height for bar with rectangular cross section (in.)

D = larger diameter for bar with circular cross section (in.), or larger height for bar with rectangular cross section (in.)

[D] = elasticity matrix

e = superscript, denotes element contribution

E = Young's modulus of elasticity (psi)

[f] = element nodal force vector (lb.)

[F] = global nodal force vector (lb.)

G = modulus of elasticity in shear (psi)

h = height of an element (in.)

i = nodal point number

I = centroidal moment of inertia (in.^{4})

J = polar moment of inertia (in.^{4})

[k] = element stiffness matrix

ℓ = axial length (in.)

M = bending moment (in.lb.)

N = shape function

P = axial tension (lb.)

r = fillet radius (in.), cylindrical coordinate

r,z,θ = cylindrical coordinates

T = torque (in.lb.)

u = axial displacement (in.)

v = radial displacement (in.)

x,y,z = rectangular coordinates

$[\alpha]$ = displacement vector

γ = shear strain (in./in.)

δ = elongation (in.)

δu = virtual displacement

δW = virtual work

Δ/D = stiffness correction parameter

ε = longitudinal strain (in./in.)

θ = angle of twist (rad.); cylindrical coordinate

κ = curvature

ν = Poisson's ratio

ξ,η = normalized coordinates

σ = normal stress (psi)

τ = shear stress (psi)

ϕ = bending angle (rad.)

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The guidance and patience of Professor Robert E. Newton are sincerely appreciated. The successful completion of this study is due largely to his timely advice.

A special thank you to my wife, Christie, for her encouragement, assistance, and sacrifices.

I. INTRODUCTION

There are many mechanical and structural applications of bars and beams which have abrupt changes in cross-sectional area. In the design of these members their deflections under expected loads must be considered. If accuracy is not essential, the deflections can be determined by relatively basic calculations. These calculations involve formulas based on the simple stress distributions found in constant section members. They do not take into consideration the reduced stiffness due to stress flow irregularities in the vicinity of the step. A stress concentration is said to exist at this location.

Through the numerical analysis technique called the finite element method, the deflection of stepped bars or beams can be accurately determined. This method is sensitive to the effect of the stress concentration.

The purpose of this study was to analyze the overall stiffness of stepped bars. An additional goal was to determine a correction parameter which could be used in design to compensate for the reduced stiffness due to the stress concentration. Finite element analyses were performed for stepped bars of various dimensions under axial, bending, and torsional loads. The resulting deflections were determined, converted to a common correction parameter, and compared.

The recent emergence of the finite element method along with the digital computer has made feasible the previously prohibitive calculations necessary for stiffness analysis. The only prior study of stiffness of stepped bars that this author could identify was accomplished by F. P. Porter [1][1]. His study of torsional vibration of irregular shafts included stiffness calculations based on the mathematical theory of elasticity using an approximate graphical construction.

[1]Numbers in brackets refer to the List of References, p. 80.

II. ELEMENTARY STRESS RELATIONSHIPS FOR THE THREE LOADING CASES

A. BASIC ASSUMPTIONS

For each of the three loading cases (axial, bending, and torsional), the stepped bar was assumed to be perfectly elastic, homogeneous, isotropic, and to exhibit a linear stress-strain relationship. In addition, it was assumed that the bar was initially straight and all loads were applied smoothly to avoid the effect of impact loading.

B. THE AXIAL LOAD CASE

A bar subjected to equal and opposite tensile loads will deform according to the equation

$$\delta = \frac{P\ell}{AE} \tag{1}$$

where δ is the elongation in length ℓ, P is the axial tension, A is the cross-sectional area, and E is the modulus of elasticity of the bar material. This equation is valid provided that the loading is applied uniformly across both ends of the bar and that the bar is of constant cross-section.

For a bar of non-uniform cross section, the elongation is estimated by dividing the bar into segments of constant cross section and adding the segment elongations. This procedure does not take into account the reduced stiffness due to the stress concentration located at the change of cross section.

Figure 1 is a longitudinal section of the bar studied. The estimated elongation of the bar is therefore

$$\delta = \frac{P}{E} \left[\frac{\ell_1}{A_1} + \frac{\ell_2}{A_2} \right] . \qquad (2)$$

A_1 and A_2 are the respective cross-sectional areas.

The axial load case was analyzed for both the plane stress state and the axisymmetric stress state. In the plane stress analysis, the bar was considered to have unit

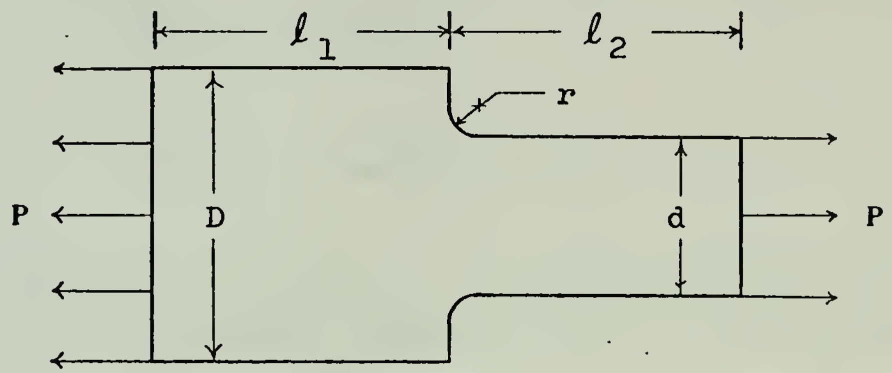


Figure 1. Geometry of stepped bar.

thickness, therefore the cross-sectional area equaled the height of the bar. In the axisymmetric stress analysis the bar was considered to be of circular cross section.

C. THE BENDING CASE

For bars subjected to flexure, or pure bending, a measure of the beam flexibility is the angle through which the ends of the bar rotate. The total angle can be determined from the equation

$$\phi = \frac{M\ell}{EI} \qquad (3)$$

where ϕ is the sum of angles ϕ_1 and ϕ_2 as described in Fig. 2. M is the bending moment and I is the moment of inertia about the horizontal centroidal axis of the cross-sectional

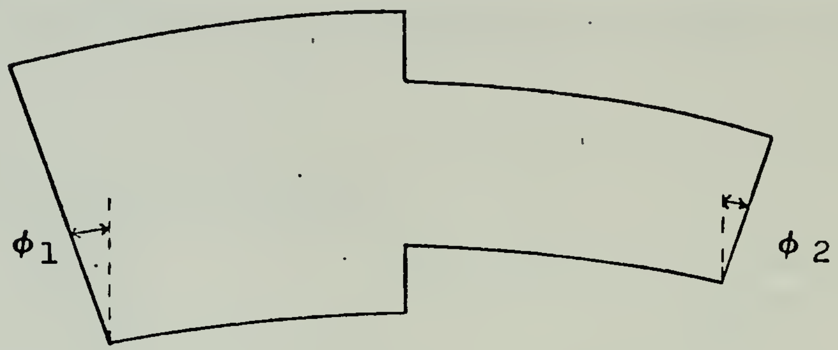


Figure 2. Bending angles.

area. Equation (3) is derived from the following equation which relates the bending moment around the neutral axis of an elastic beam to the curvature, κ, of the elastic curve.

$$\kappa = \frac{M}{EI} .$$

This theory is valid provided:

1. Plane sections remain plane during bending.
2. The bar is of constant cross section.
3. Bending is such that shear strains are negligible.

As in the axial load case, the stepped bar can be considered in two sections, each of constant cross section. The estimate for the total bending angle is therefore

$$\phi = \frac{M}{E} \left[\frac{\ell_1}{I_1} + \frac{\ell_2}{I_2} \right]. \tag{4}$$

The bar analyzed was of rectangular cross section and considered to have unit thickness. The bending moment was applied by a linearly varying normal stress across both ends of the bar exerting tension on one side of the neutral surface and compression on the other side as shown in Figure 3.

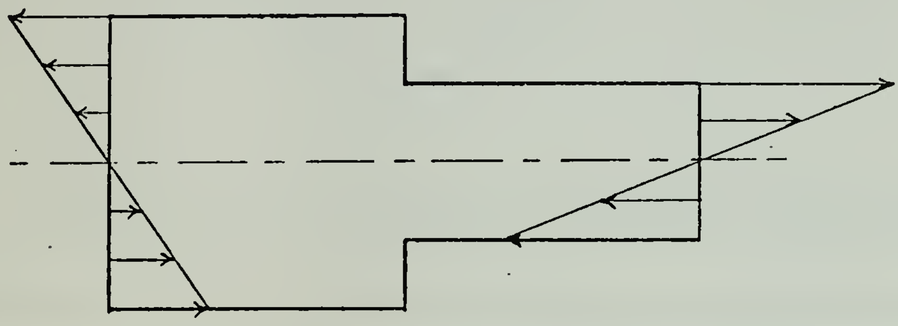


Figure 3. Bending stress distribution.

D. THE TORSION CASE

The equation for angular deflection of a uniform, round, cross-section bar subjected to torsion is

$$\theta = \frac{T\ell}{JG} \quad . \tag{5}$$

θ is the angular deflection, T is the torque, J is the polar moment of inertia of the cross-sectional area, and G is the modulus of elasticity in shear. It is assumed that plane sections perpendicular to the axis of the bar remain plane and that a radial line remains straight when the bar is twisted.

In this study, one end of the bar was held fixed and a torque was applied to the other end such that the shear

stress varied linearly from zero at the centerline to a maximum at the outer radius as shown in Fig. 4.

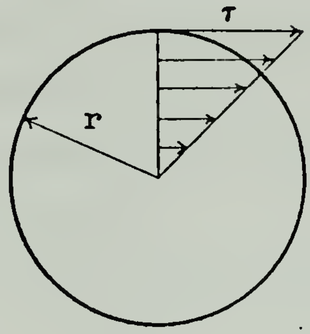


Figure 4. Torsional stress applied to circular cross section.

As before, the bar was considered in two sections of constant cross section and the total angle of twist was the sum of the angles for each section. Therefore the estimate for the entire bar is

$$\theta = \frac{T}{G} \left[\frac{\ell_1}{J_1} + \frac{\ell_2}{J_2} \right] . \qquad (6)$$

III. METHOD OF STIFFNESS COMPARISON

It is well documented that the deflection of a bar of non-uniform cross section is slightly greater than would be indicated by basic equations (2), (4), and (6). This decreased stiffness is caused by the stress intensification at the point of discontinuity.

One way of accounting for this decreased stiffness is to consider the effect of the stress intensification on the bar as a whole to be the same as the effect of moving the location of the discontinuity. Increasing the length of the smaller portion of the bar and decreasing the length of the larger portion by the same amount would achieve the same effect on its stiffness.

This apparent change in the location of the bar discontinuity was calculated and used as a measure to compare stiffness of bars of different dimensions. The size of this change was defined as the variable "Δ" and is illustrated in Fig. 5.

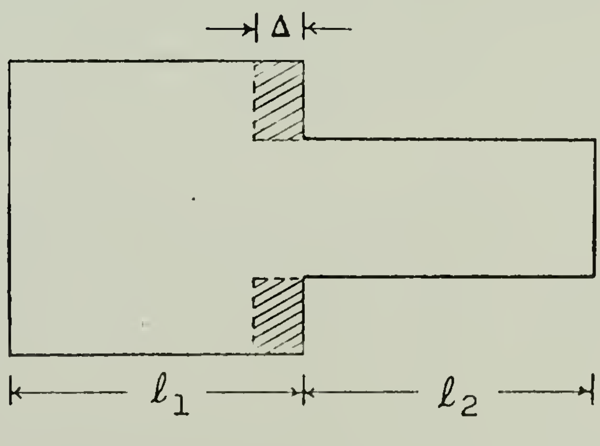


Figure 5. Geometric interpretation of stiffness correction parameter.

To calculate Δ, equations (2), (4), and (6) were altered as follows:

$$\delta = \frac{P}{E}\left[\frac{\ell_1-\Delta}{A_1} + \frac{\ell_2+\Delta}{A_2}\right] \tag{7}$$

$$\phi = \frac{M}{E}\left[\frac{\ell_1-\Delta}{I_1} + \frac{\ell_2+\Delta}{I_2}\right] \tag{8}$$

$$\theta = \frac{T}{G}\left[\frac{\ell_1-\Delta}{J_1} + \frac{\ell_2+\Delta}{J_2}\right]. \tag{9}$$

In each loading case, the deflections required to calculate δ, ϕ, and θ were determined by digital computer finite element stress analysis. Knowing the load, modulus of elasticity, and physical dimensions of the bar, the parameter Δ was determined. The Δ parameter is common to equations (7), (8), and (9) and therefore is a means of comparing stiffness corrections for the three types of loading. In addition, it is independent of the load applied because the magnitude of the deflection and the load applied are directly proportional.

Since the Δ parameter is a means of correcting for the deflection of a stepped bar under load, it was defined as the "stiffness correction parameter."

IV. SOFTWARE

A. BACKGROUND

Before the stiffness correction parameter could be determined, the deflections of the bar under each of the three loading conditions had to be determined. Although photoelastic analysis and direct strain measurement are suitable for determining the effect of stress concentration, the ideal method for overall stiffness evaluation is the finite element stress analysis.

In very brief and general terms, the finite element method is an advanced numerical method that has been made possible by the high speed digital computer. When applied to structures it can be used to perform two or three dimensional stress analyses. The structure under study is divided into subdivisions called finite elements. Adjacent elements are connected along the entire inter-element boundary. Specific points at regular intervals along the element boundaries are designated and defined as nodal points or nodes. When a load is applied to the structure, the displacements of each of these nodal points are calculated and used to determine the stresses.

For each of the problems, maximum use of existing computer programs was made. For the plane stress analysis of the axial and bending load cases, a program titled PLISOP [2] was used. It was developed at the Naval Postgraduate School, Monterey, California and has been in use for several

years. For the torsional loading case, a program titled TORT 2 [3] was used. It was developed at the University of Wales, Swansea, and had been installed locally but had not been used or completely debugged. In order to study the fourth problem, the axially loaded bar of circular cross section, the capability of PLISOP had to be expanded to perform an axisymmetric stress analysis.

B. MESH GENERATION

The network of lines describing the division of a structure into finite elements is referred to as a mesh. Normally a finer mesh more accurately describes the physical structure under study and results in a more accurate stress analysis. Unfortunately, increasing the number of subdivisions or elements greatly increases the computational effort involved in the problem solution. As a result, the maximum capabilities of most general purpose digital computers are rapidly reached as meshes are refined.

In many stress analyses, the area of interest is only a small portion of the structure. In these cases the mesh can be very fine in the area of interest and much coarser for the rest of the structure. This minimizes the total number of elements yet achieves accuracy where it is desired. In this study, however, the analysis of the structure as a whole was of interest. Therefore it was desirable to obtain a mesh as uniformly refined as possible within computer program limitations. The only practical means of maximizing accuracy yet minimizing the total number of elements was to

take advantage of symmetry. This was done by dividing the bar in half along the longitudinal axis.

It was found satisfactory to take the lengths ℓ_1, ℓ_2 of the two bar segments equal to the larger height dimension D. This provided a nearly undisturbed stress distribution at both ends of the bar. Thus the stress intensification resulting from the abrupt change of cross section is negligible at the ends of the bar.

Based on these considerations, the stepped bar was represented as shown in Fig. 6. In each loading case the ratio of d/D and the fillet radius were varied. These variations necessitated minor changes in the mesh representation, however, this general discussion will refer to the arrangement of Fig. 6.

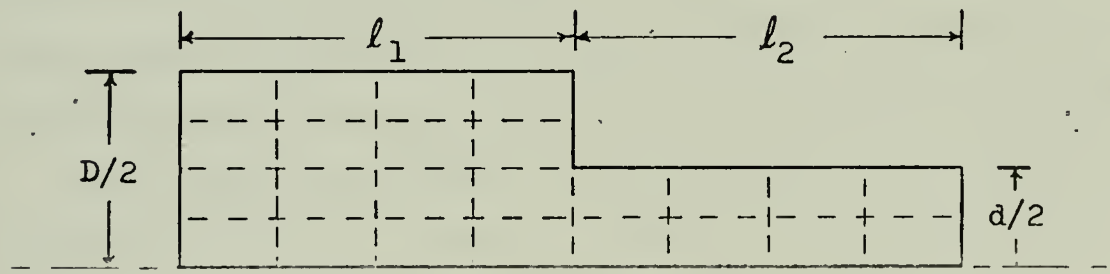


Figure 6. Finite element mesh.

For this basic geometry, the mesh used consisted of twenty-four isoparametric quadrilateral elements. The concept of the isoparametric element requires lengthy explanation and is not considered essential to this development. The reader is referred to chapter eight of Ref. 4, for the background of the isoparametric element. Quadrilateral

elements were used as opposed to triangular elements. Both of the computer programs used isoparametric quadrilateral elements. Such elements can be linear, quadratic, or cubic. In this study, the only curved portion of the mesh, the fillet, was adequately described by quadratic elements.

The quadratic, isoparametric, quadrilateral element has four corner nodal points, or nodes, and four mid-side nodes. The coordinates of each of these nodes are part of the input data for both the PLISOP and the TORT 2 programs. The task of providing this coordinate data as well as data to describe how the elements fit together is a considerable one. Fortunately, another computer program was developed to perform just this task. This program, titled PLIMEG [5], generates a mesh as specified. It requires only enough geometric data to describe the overall area to be discretized. The directions for use of PLIMEG are contained in the program.

The twenty-four element mesh was chosen as an adequate representation of the region. A later convergence study, which used a finer mesh, confirmed the adequacy of the twenty-four element mesh for this primarily comparative study.

Illustrations of the meshes used for the various geometries are shown in Appendix A.

C. COMPUTER PROGRAM EMPLOYED

1. The PLISOP program performs a plane stress or a plane strain analysis by the finite element method. Detailed

directions for its use are contained in the program. The bulk of the input data can be prepared by the mesh generating program, PLIMEG, previously mentioned. The axial and bending effects were achieved by the loads applied and by the boundary conditions imposed.

The load vectors must be derived externally and are applied by specifically defining the load and its direction at particular nodal points in the mesh. Derivation of the force vectors for both axial and bending loading is shown in Appendix B, sections 1 and 2, respectively.

The boundary conditions necessary for the axial loading of the bar half-section shown in Fig. 6 consisted of constraining each node along the bottom surface (bar neutral surface) to freedom of movement in the longitudinal direction only. In addition, one of the corner nodes along this surface was pinned to prevent longitudinal motion as well. For the bending load, the nodes along the bottom surface were free to move in the vertical direction only. The two corner nodes along this surface were pinned to allow only rotational displacement at these two points.

2. The TORT 2 program performs a stress analysis of circular sectioned solids with varying diameter using the finite element method. The directions for its use are contained in a separate booklet accompanying the program. The same meshes were used to represent the stepped bar except the origin of the coordinate system had to be at the lower right corner of the bar. The origin for the mesh used previously for the PLISOP program was at the lower

left corner. A relatively simple computer routine can be used to renumber the nodal point data provided by the PLIMEG program, but the connectivity data was revised by hand.

Loading is applied by specifically assigning values of shear stress at particular nodal points.

3. To study a bar of circular cross section under axial load, an axisymmetric stress analysis capability was required. The most direct way of obtaining this capability was to alter the PLISOP program. Due to the symmetry of a body of revolution, the problem remains mathematically two dimensional. The principal difference between the plane stress and the axisymmetric stress problems is the introduction of a fourth component of stress, the "hoop" stress. The details of the development of the axisymmetric capability are included in Appendix C.

Since the PLISOP program is essentially unchanged by this added capability, the directions for its use are the same. Appendix D contains notes on the axisymmetric capability intended to amplify the directions contained in the program. Appendix E contains a listing of the PLISOP program in its revised form.

As in the plane stress analyses of the axial and bending load cases, the load vectors must also be derived externally for the axisymmetric stress analysis. This derivation is included in Appendix B, section 3.

V. RESULTS

A. PRESENTATION OF RESULTS

For each of the loading cases, nine variations of the bar dimensions were analyzed. The dimensions and results were nondimensionalized by dividing each by the dimension, D, which was held constant throughout the study. The values of the stiffness correction parameter, Δ/D, for each of the four problems are presented in tabular form, Tables I - IV, and graphically, Figs. 7 - 10.

From the graphical presentations it is evident that the largest correction parameters are required when there is no fillet. As the fillet radius is increased, Δ/D decreases. In addition, as the dimension, d, is varied with r held constant, the largest correction is required for the ratio of d/D equal to one-half.

The negative values of Δ/D indicate the bar is stiffer than equations (2), (4), and (6) would show. This is explained by the fact that these equations, as applied, do not take into consideration the increased cross-sectional area resulting from the fillet. The fillet area was not included in the equations for calculating the correction in order to keep the calculations simple and uniform for all cases.

B. COMPARISON OF THE FOUR CASES

In order to compare the four cases, a plot of Δ/D versus r/D was prepared for each of the three ratios of d/D. Figures

Table I

Δ/D for Axial Load
(rectangular cross section)

r/D \ d/D	0.25	0.50	0.75
0.0	0.19	0.22	0.16
0.125	0.12	0.15	0.09
0.250	0.04	0.06	0.0

Table II

Δ/D for Axial Load
(circular cross section)

r/D \ d/D	0.25	0.50	0.75
0.0	0.10	0.20	0.12
0.125	0.10	0.13	0.05
0.250	-0.05	0.04	-0.04

Table III

Δ/D for Bending
(rectangular cross section)

r/D \ d/D	0.25	0.50	0.75
0.0	0.07	0.12	0.11
0.125	0.0	0.06	0.04
0.250	-0.09	-0.04	-0.05

Table IV

Δ/D for Torsion
(circular cross section)

r/D \ d/D	0.25	0.50	0.75
0.0	0.03	0.06	0.06
0.125	-0.04	0.0	-0.01
0.250	-0.12	-0.09	-0.09

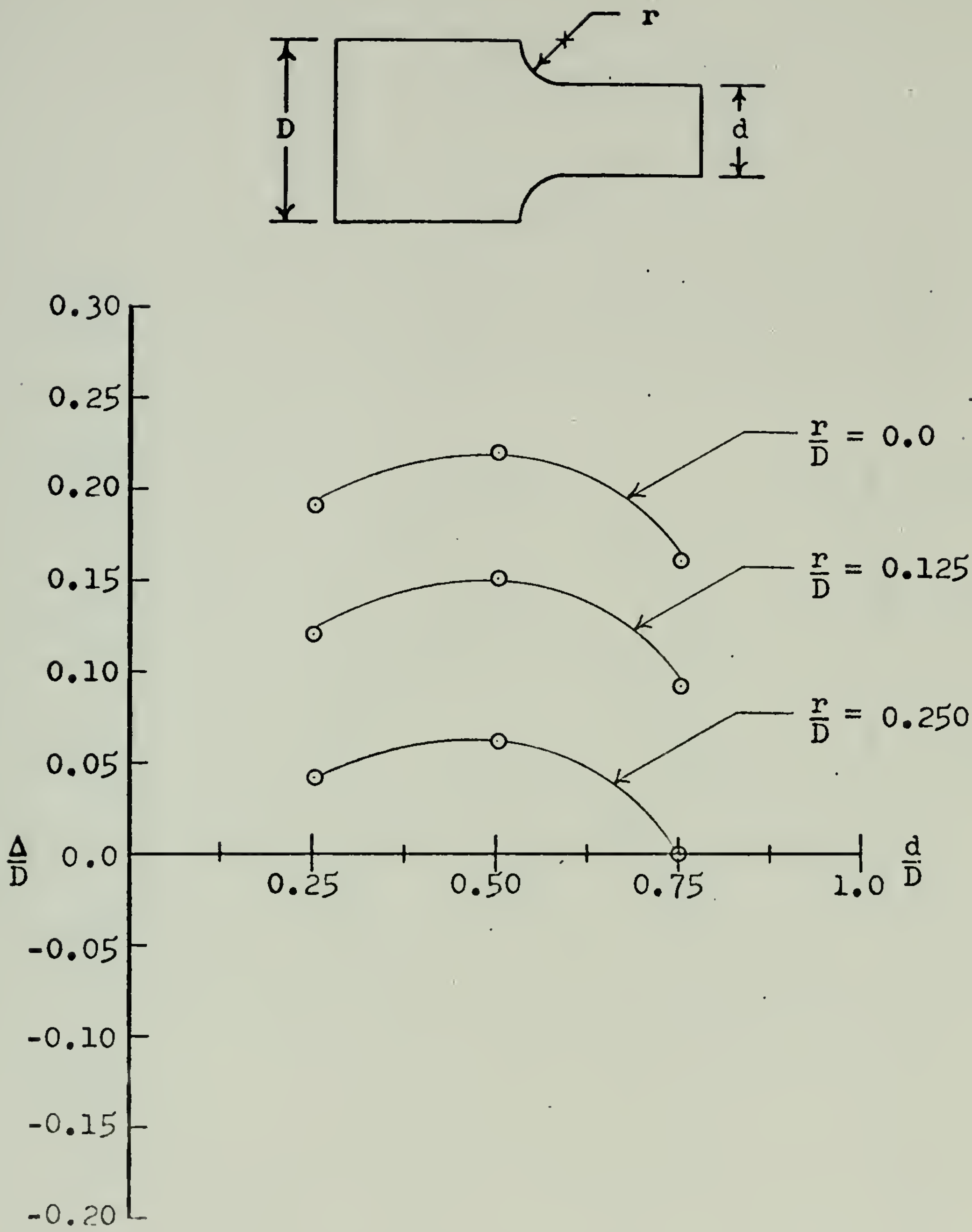


Figure 7. Stiffness correction for axial load (rectangular cross section).

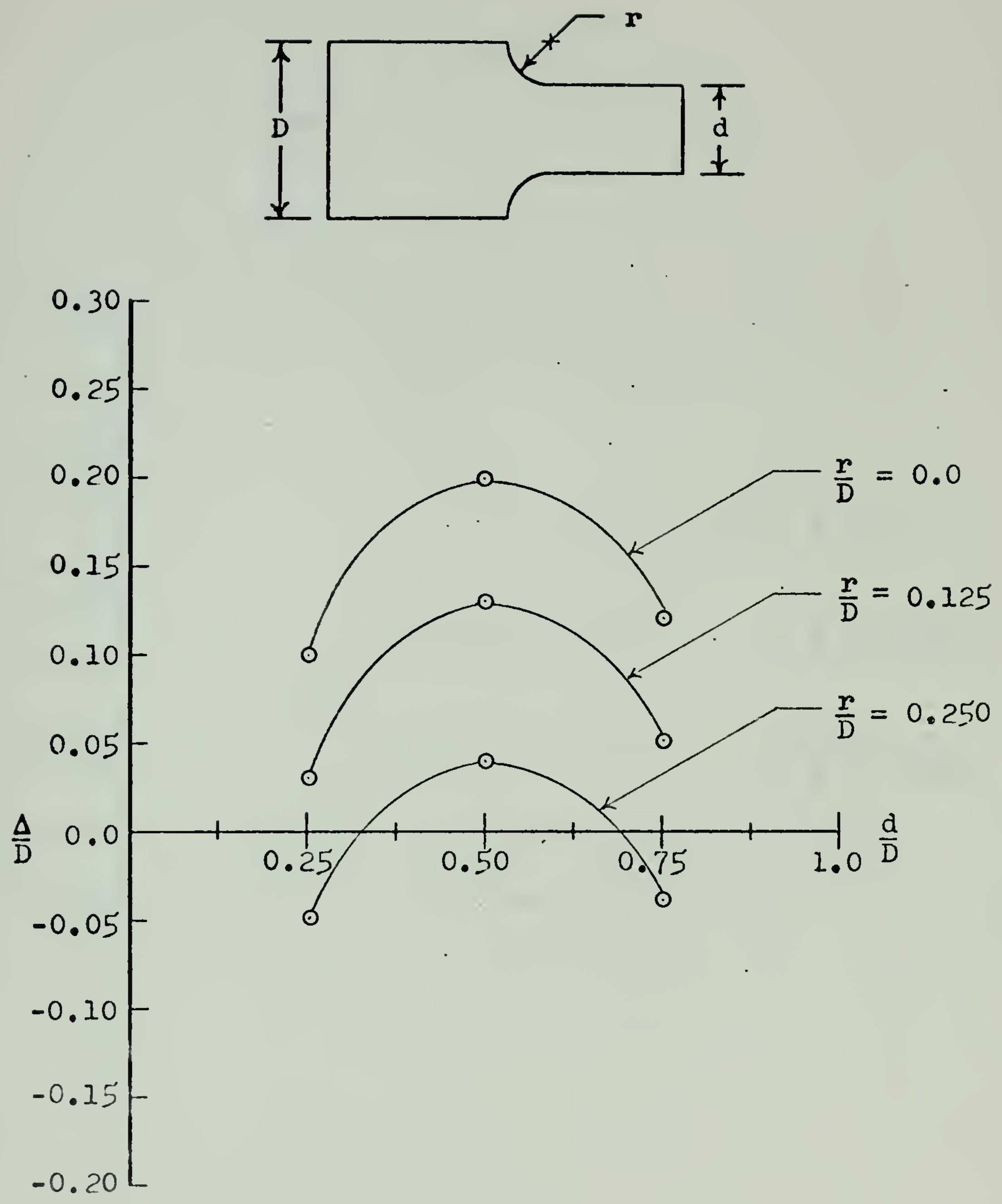


Figure 8. Stiffness correction for axial load (circular cross section).

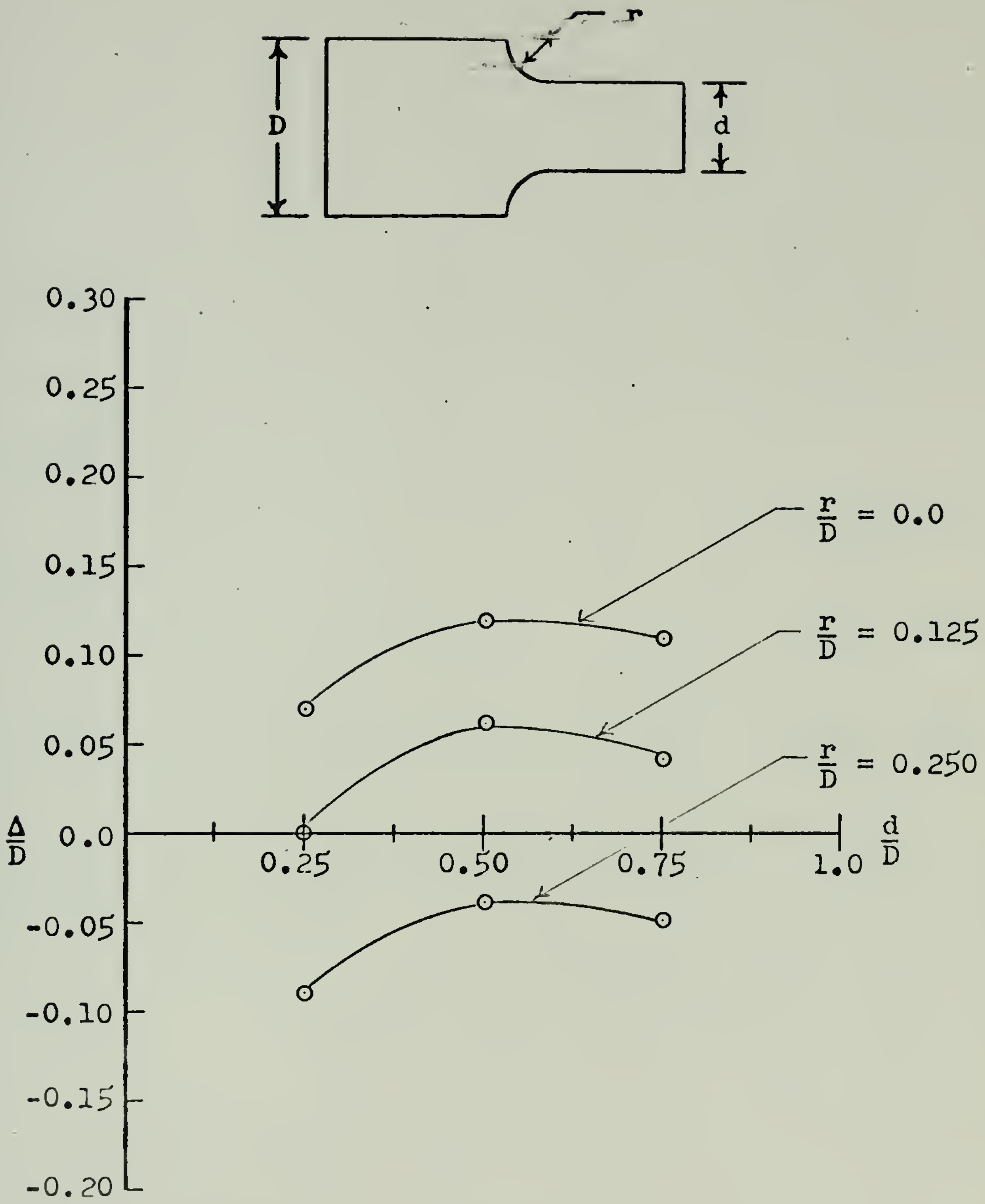


Figure 9. Stiffness correction for bending (rectangular cross section).

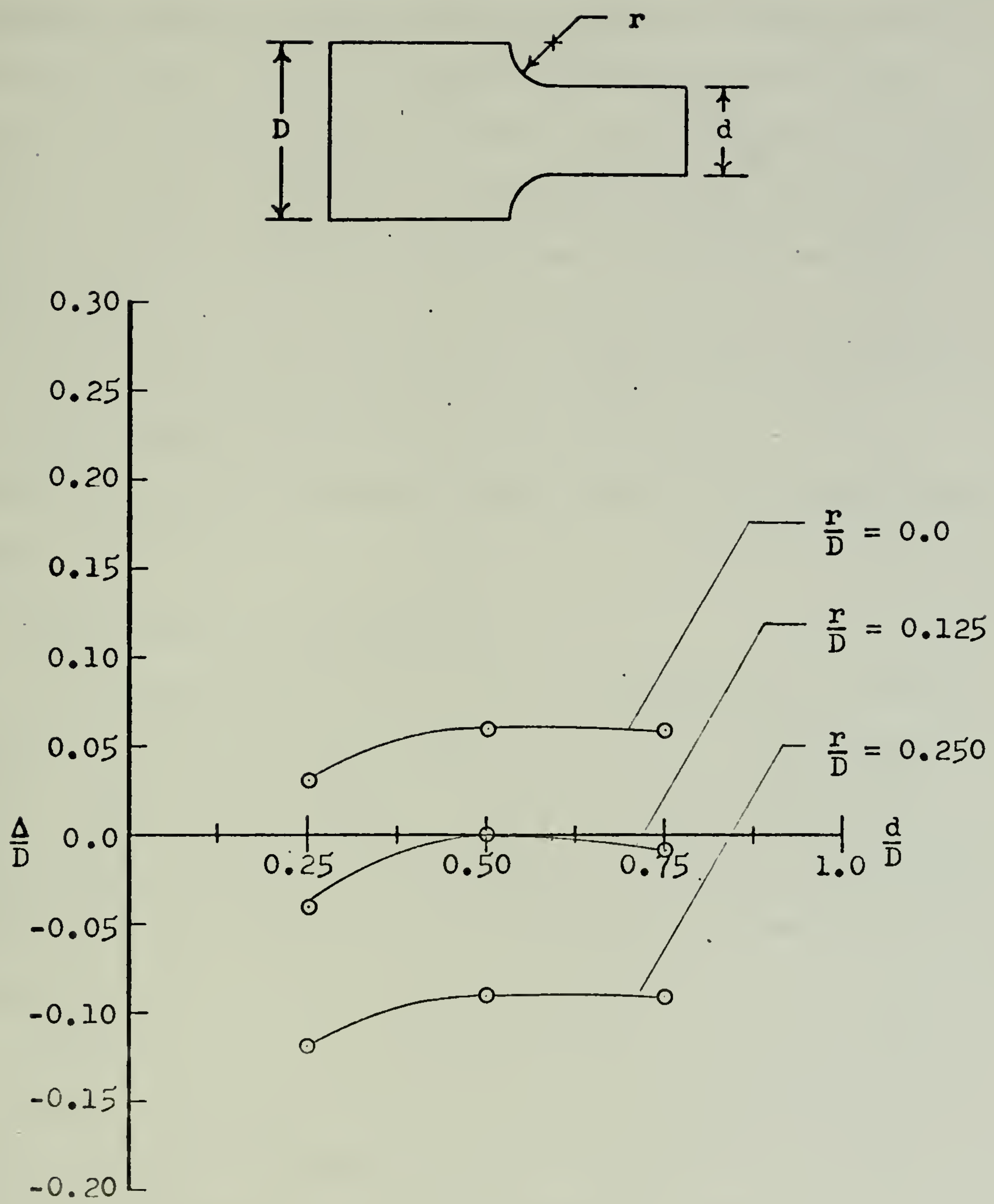


Figure 10. Stiffness correction for torsion (circular cross section).

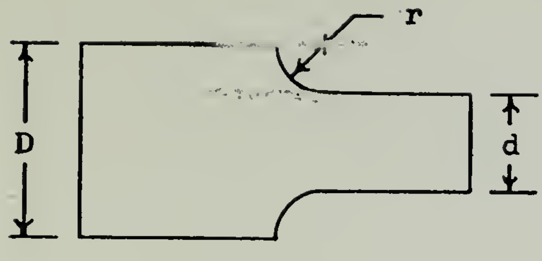
11, 12, and 13 show the effect on stiffness of the three different types of loading and two types of cross section. The most severe type of loading is clearly axial as indicated by the larger values of Δ/D. I. M. Allison [6] also found the axial loading to be the most severe of the three. His conclusions, however, were based on stress concentration factors.

Comparison of the two types of cross section subjected to axial load indicates the circular cross section is stiffer than the rectangular cross section for a particular ratio of d/D.

C. CONVERGENCE AND UNCERTAINTY

The normal methods for proving convergence of a finite element solution are by further refining the mesh or by using higher order elements. Both of these methods were investigated. For the basic bar case (D=4, d=2, r=0) under axial load, the convergence test results are presented in Fig. 14.

These results show the improvement in accuracy gained by using the quadratic rather than the linear elements. Although it can be assumed that cubic elements would give further improvement, other considerations prevailed. Since it was desired to cover a wide range of problems, the additional computational effort required for cubic elements was deemed an unnecessary luxury. Also, since the geometry of the bar was relatively simple, consisting of plane or cylindric surfaces, cubic elements would not provide the same



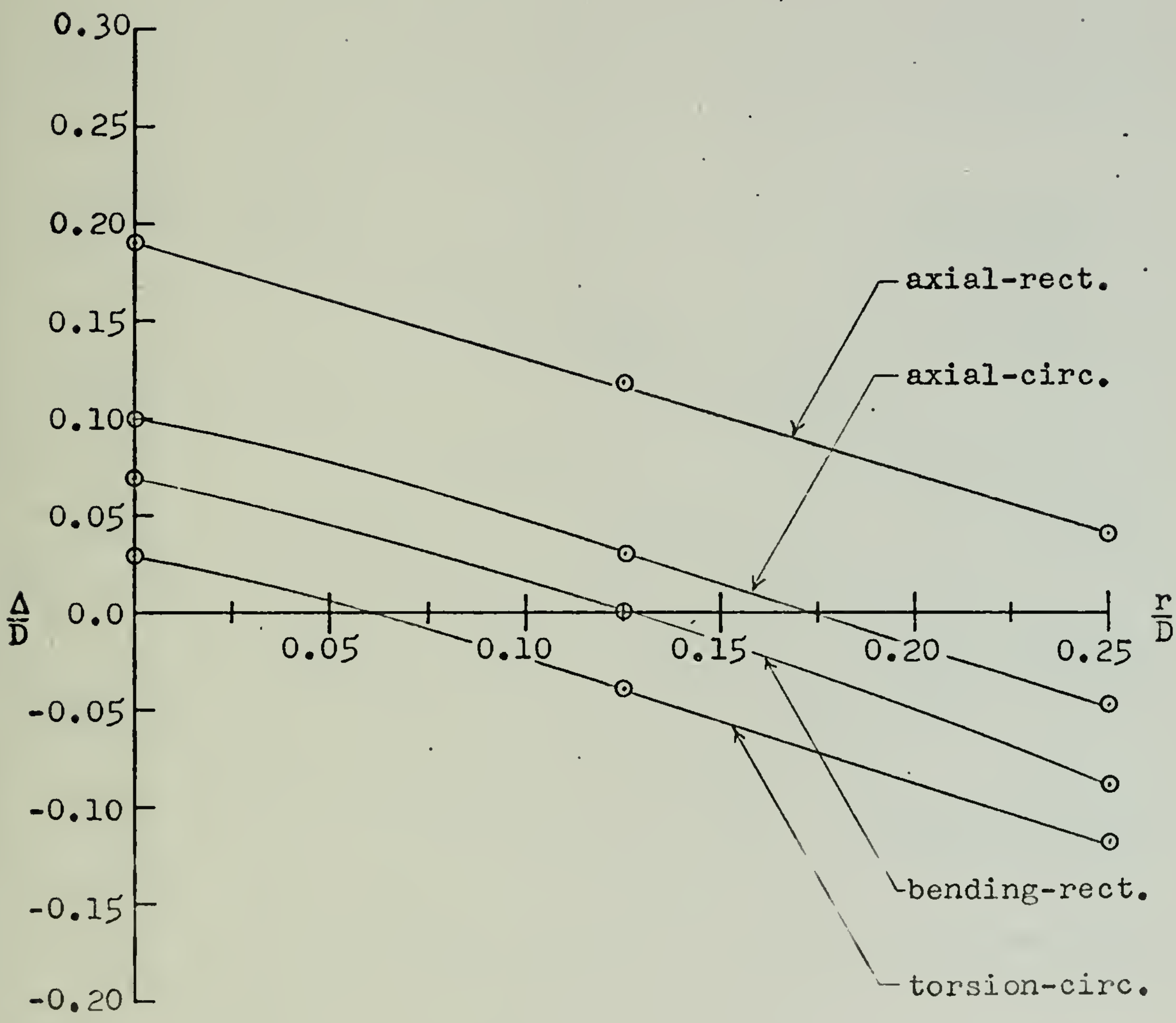


Figure 11. Stiffness correction for various loads with d/D = .25.

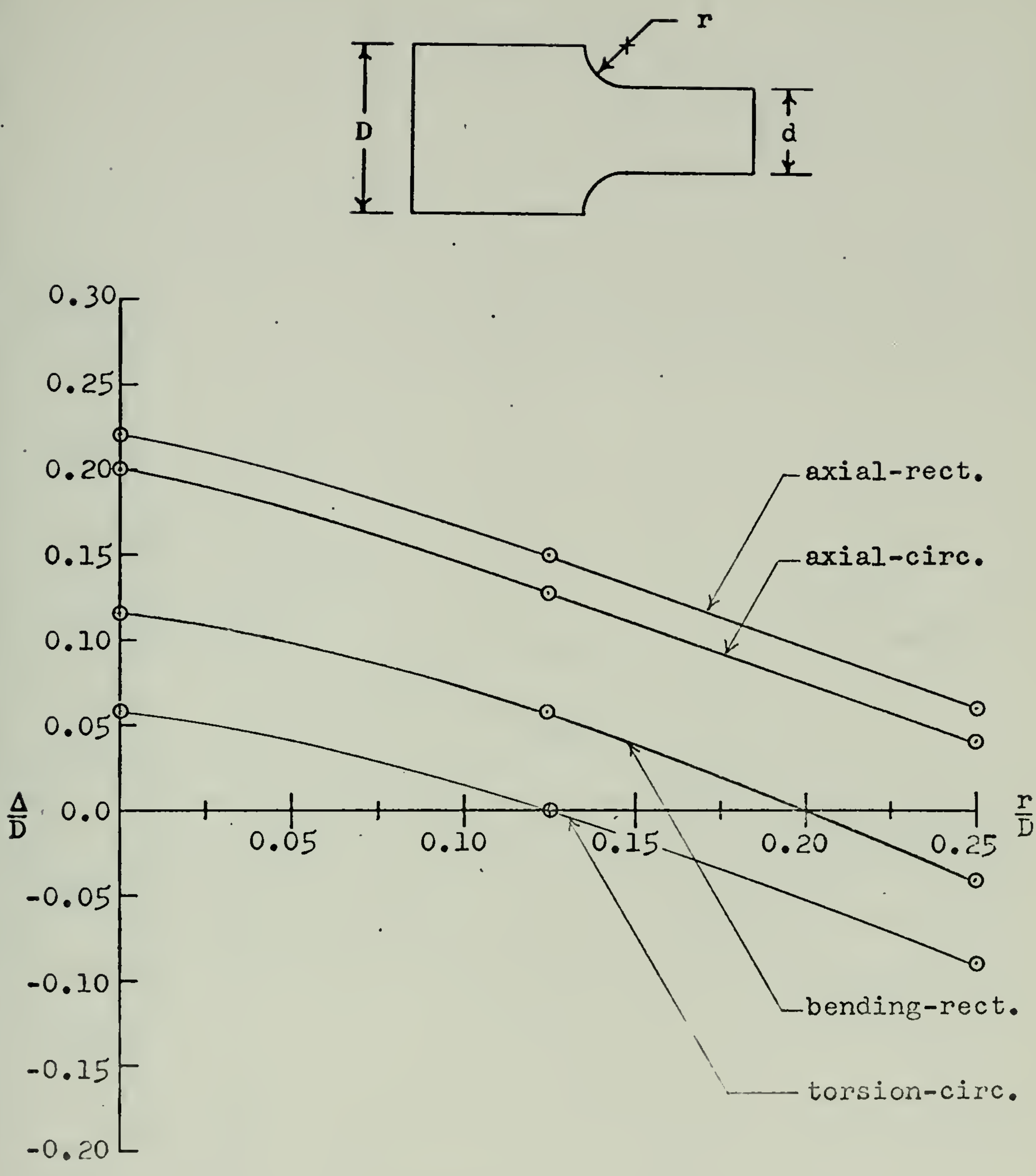


Figure 12. Stiffness correction for various loads with d/D = .50.

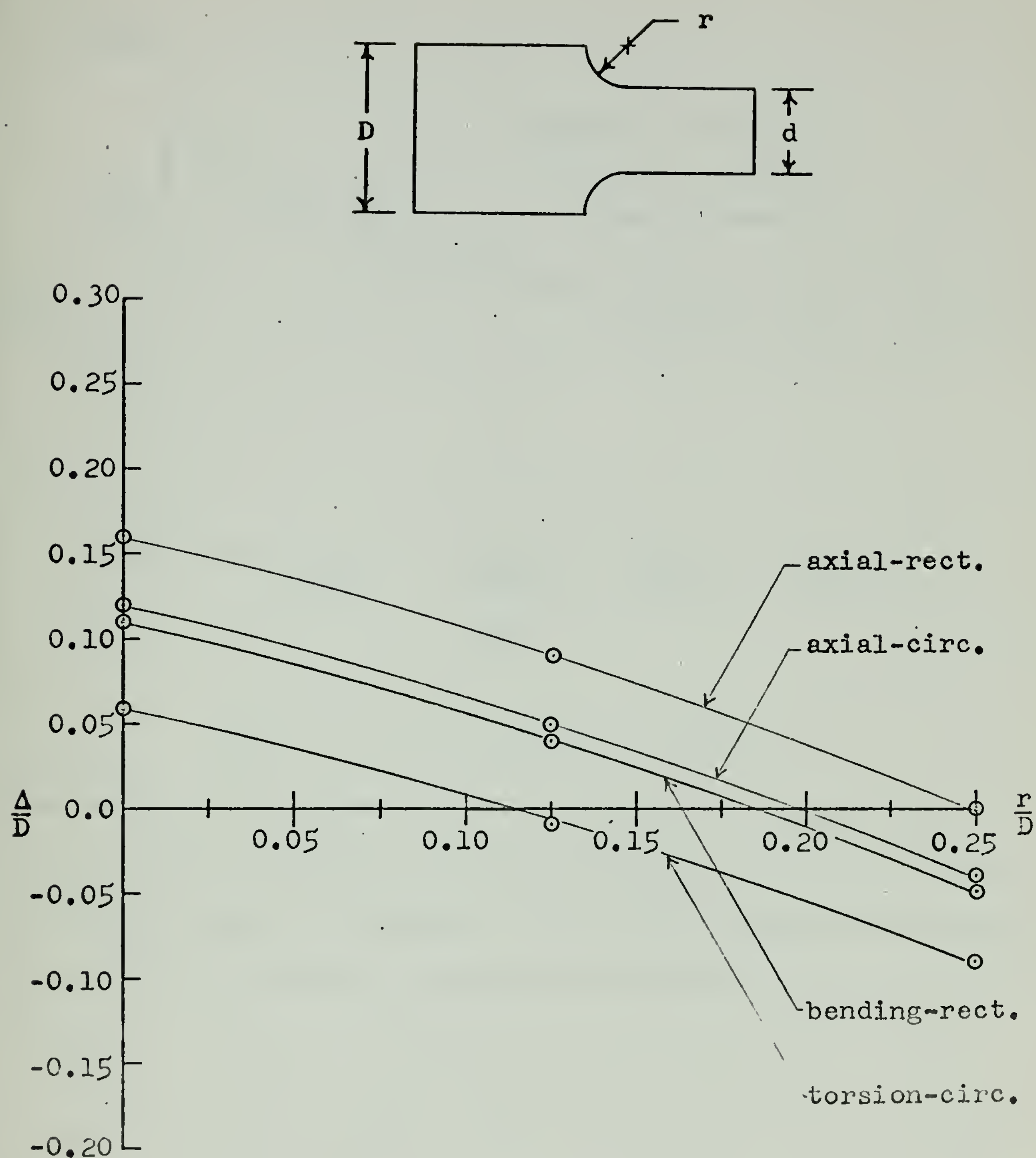


Figure 13. Stiffness correction for various loads with d/D = .75.

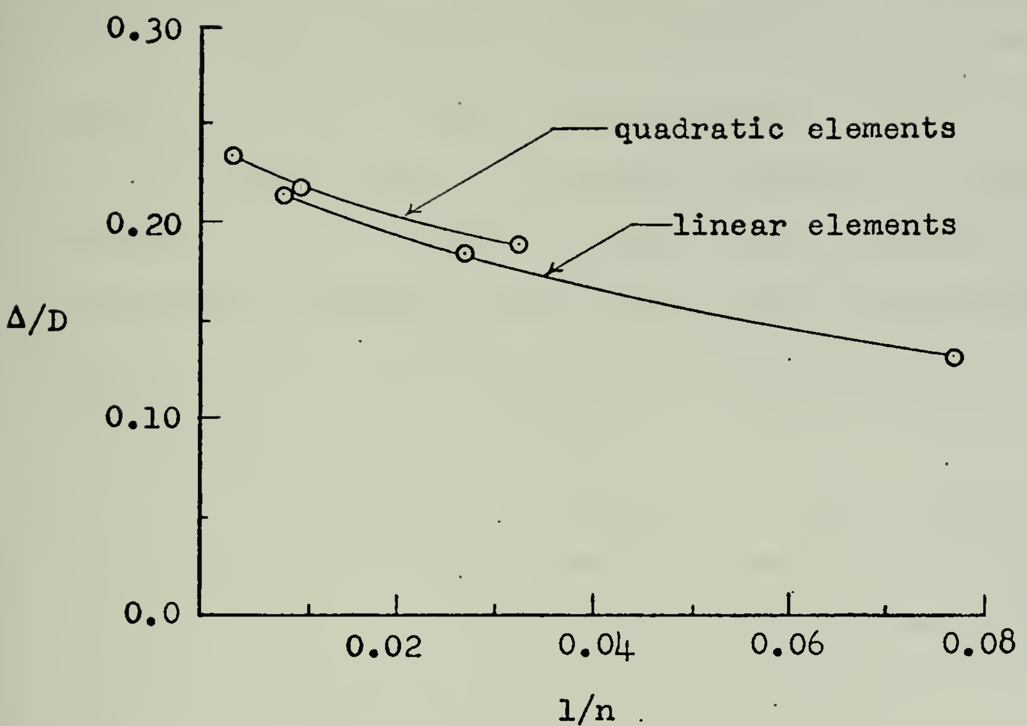


Figure 14. Δ/D vs.1/(no. of nodes).

percentage improvement that the quadratic elements did.

In order to check the accuracy achieved by mesh refinement with quadratic elements, the PLISOP program limits were altered to accommodate a ninety-six quadratic element mesh. Figure 14 indicates this mesh has come very close to convergence. Based on these results, the uncertainty in the values of Δ/D in Tables I - IV is considered to be the difference between the ninety-six element result and the twenty-four element result, rounded to the second decimal place. The uncertainty is therefore +.02.

An additional test problem was run to verify that the geometry studied, with dimensions ℓ_1 and ℓ_2 equal to the

larger height dimension, was adequate to ensure that the stress intensification at the discontinuity did not significantly affect the stress distribution at the ends of the bar. To check this aspect, the length dimensions ℓ_1 and ℓ_2 were increased to 1.5 times the larger height dimension. The resulting Δ/D parameter was within .001 of the corresponding case with the shorter length, thus confirming the sufficiency of the ratios $\ell_1/D=\ell_2/D = 1.0$.

The one geometry and loading case which can be compared with Porter's work agrees very favorably. For torsion of a bar with $d/D = 0.75$ and $r/D = 0.0$, the stiffness correction parameters agree to the third decimal place.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The stiffness correction parameters obtained are useful and applicable in the design of stepped bars. The tables and curves included in Section V can be used with confidence to the first decimal place of Δ/D within the range of physical bar dimensions studied.

The results also yield important comparative data for stepped bars of various dimensions under the three types of loading. The following comparative observations were made:

1. The value of Δ/D for stepped bars subjected to axial load is larger than the corresponding value for bending or torsion.

2. The stepped bar with ratio d/D equal to one-half yields a larger value of Δ/D than the other ratios studied.

3. Fillets reduce stress intensification at the location of the step and therefore increase the stiffness of the bar.

In addition, it was observed that the localized stress intensification had a negligible effect on the cross-sectional stress distribution at a distance equal to the height of the bar. This illustrates the St. Venant principle of rapid dissipation of localized stresses.

B. RECOMMENDATIONS

It would be of interest to isolate one loading case and through use of more refined meshes and cubic elements achieve

correction parameters accurate to the third decimal place. In addition, a great deal more comparative information could be gathered with a three dimensional finite element solution program. Both of these would require altering existing computer programs and would greatly increase the computer core storage space required for each run in this study.

APPENDIX A: FINITE ELEMENT MESHES EMPLOYED

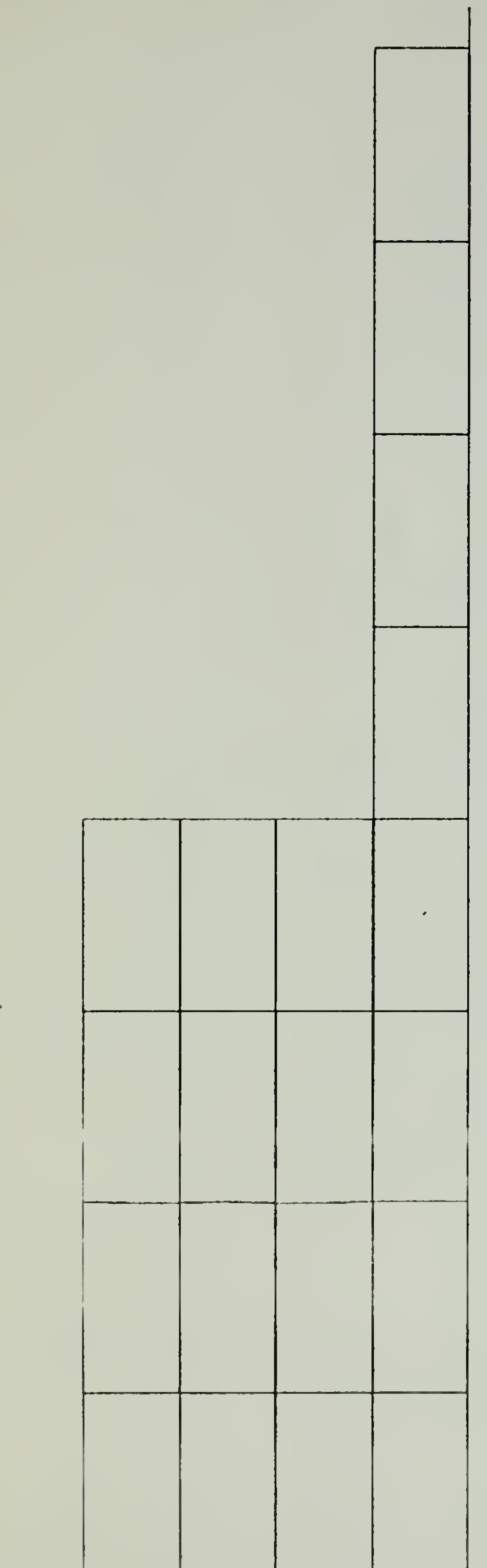


Figure 15. d/D = 0.25 r/D = 0.0

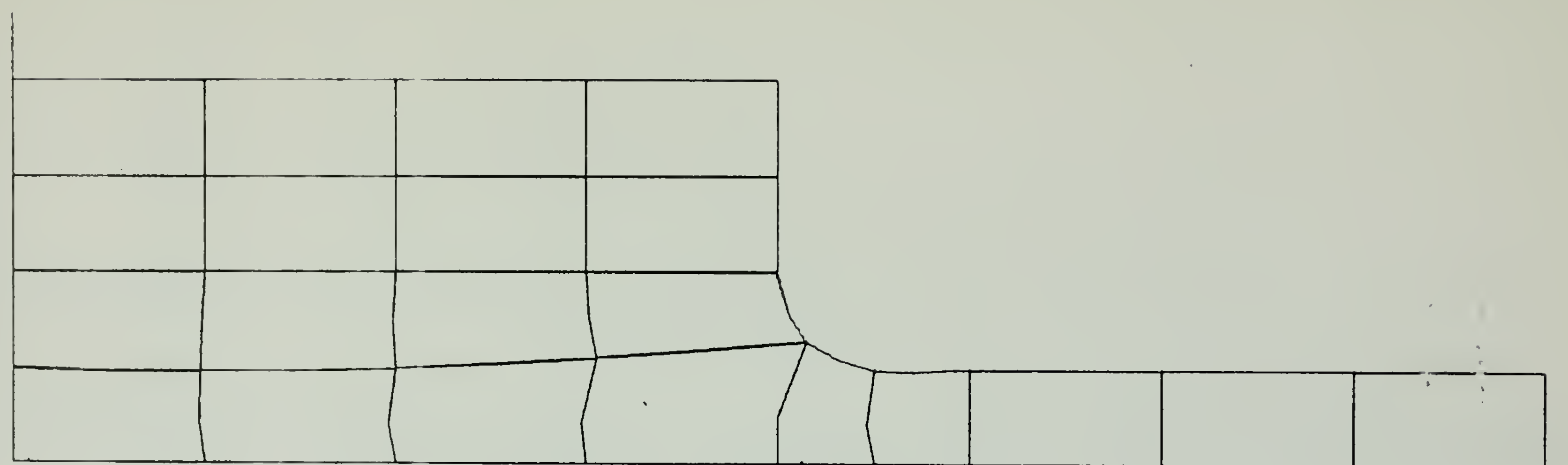


Figure 16. d/D = 0.25 r/D = 0.125

Figure 17. $d/D = 0.25$ $r/D = 0.250$

Figure 18. d/D = 0.50 r/D = 0.0

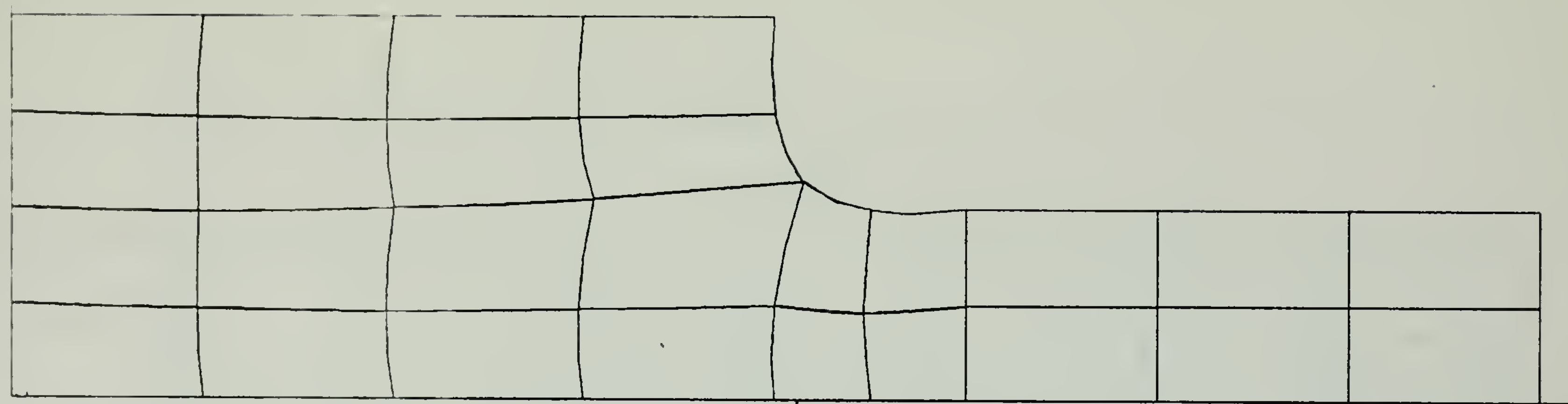


Figure 19. d/D = 0.50 r/D = 0.125

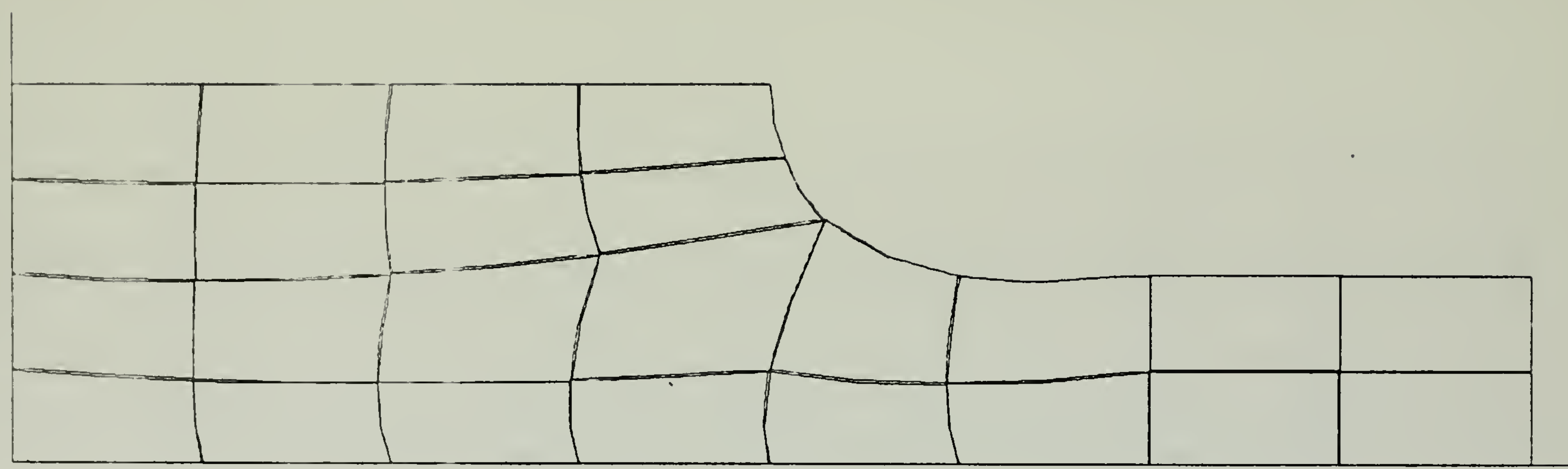


Figure 20. d/D = 0.50 r/D = 0.250

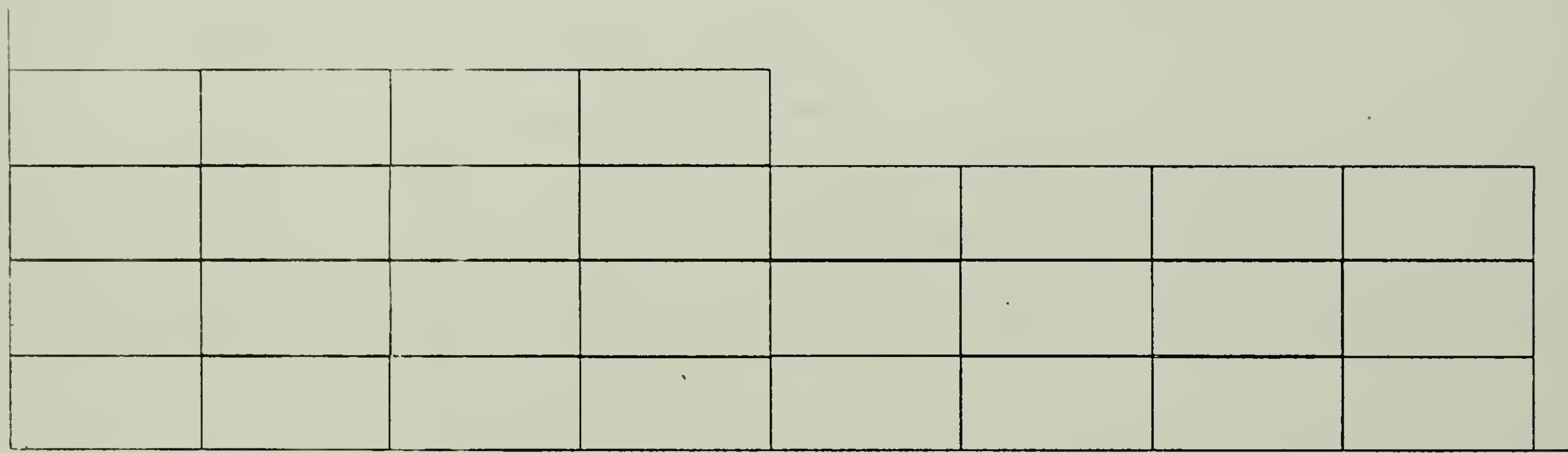


Figure 21.   d/D = 0.75   r/D = 0.0

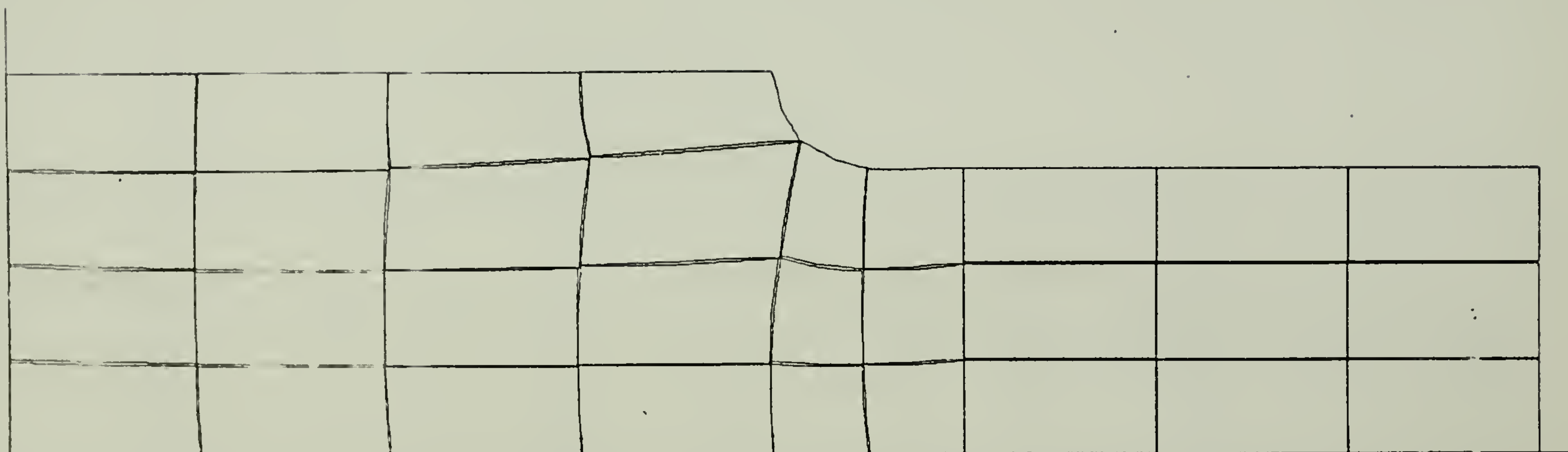


Figure 22. d/D = 0.75 r/D = 0.125

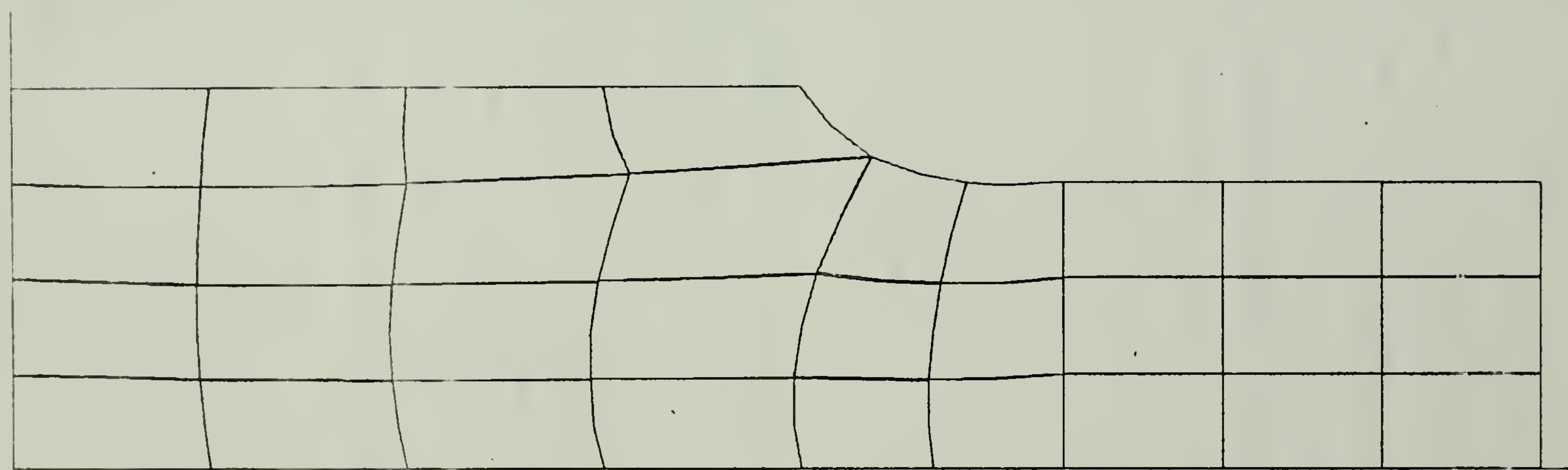


Figure 23. d/D = 0.75 r/D = 0.250

APPENDIX B: DERIVATION OF EQUIVALENT NODAL FORCES

1. Plane Stress Analysis of the Axially Loaded Bar

Boundary stresses and distributed loads are converted to equivalent nodal forces by the principle of virtual work. For the case of external stress σ applied along the boundary of an element, the nodal forces are developed as follows:

$$\delta W = [\delta u_i]^T [f_i]^e = \int \sigma \cdot \delta u \, d(\text{area})$$

substituting

$$\delta u = [\delta u_i]^T [N_i]^T$$

$$[f_i]^e = \int \sigma [N_i]^T d(\text{area}) \tag{10}$$

where

δW = increment of virtual work

f_i = nodal forces

σ = applied stress

δu_i = incremental nodal displacements

δu = the virtual displacement of a typical point on the element surface

N_i = nodal shape functions

e = superscript, denotes element contribution.

The quadratic, isoparametric, quadrilateral element pictured below was used throughout this study. The shape functions are expressed in terms of the local, normalized coordinates (ξ, η). For loading as shown, the nodal force equations become:

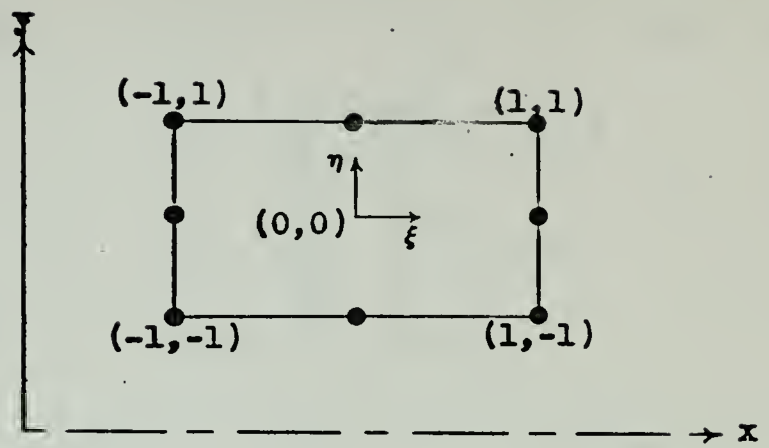


Figure 24. Quadratic, isoparametric, quadrilateral element.

$$\begin{bmatrix} f_3 \\ f_2 \\ f_1 \end{bmatrix} = \int_{-1}^{+1} \sigma \cdot \frac{h}{2} \cdot \begin{bmatrix} N_3 \\ N_2 \\ N_1 \end{bmatrix} d\eta$$

where

h = height of the element

d(area) = dy = (h/2)dη for bar of unit thickness.

Substituting the shape functions and integrating, the result for σ equal constant is:

$$\begin{bmatrix} f_3 \\ f_2 \\ f_1 \end{bmatrix} = \frac{\sigma h}{2} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \qquad (11)$$

Where nodes are shared by two adjacent elements, the nodal forces are summed.

2. Plane Stress Analysis of a Bar Under Bending Load

The equivalent nodal forces for the linearly varying stress were derived in the same manner as the axially loaded

case except for the applied stress. The linearly varying stress shown on the element surface, shown below, can be

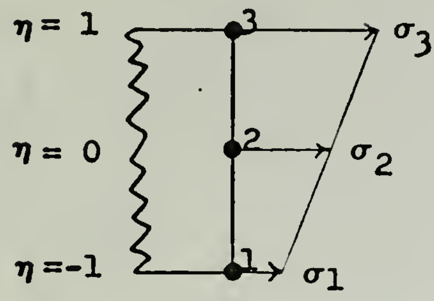


Figure 25. Elemental linearly varying stress distribution.

represented by the equation

$$\sigma = \sigma_1\left(\frac{1-\eta}{2}\right) + \sigma_3\left(\frac{1+\eta}{2}\right). \tag{12}$$

Substituting into the equation for nodal forces,

$$\begin{bmatrix} f_3 \\ f_2 \\ f_1 \end{bmatrix} = \int_{-1}^{+1} \left\{\sigma_1\left(\frac{1-\eta}{2}\right) + \sigma_3\left(\frac{1+\eta}{2}\right)\right\}\cdot\frac{h}{2}\cdot\begin{bmatrix} N_3 \\ N_2 \\ N_1 \end{bmatrix} d\eta$$

$$\begin{bmatrix} f_3 \\ f_2 \\ f_1 \end{bmatrix} = \frac{h}{6}\begin{bmatrix} \sigma_3 \\ 2\sigma_1 + 2\sigma_3 \\ \sigma_1 \end{bmatrix}. \tag{13}$$

Applying this formula to one end of a particular bar spanned by two elements as shown in Fig. 26 and letting $\sigma_3^1 = \sigma$, then $\sigma_1^1 = \sigma_3^2 = \sigma/2$ and $\sigma_1^2 = 0$.

Therefore, the nodal forces along this surface, numbered from top to bottom are:

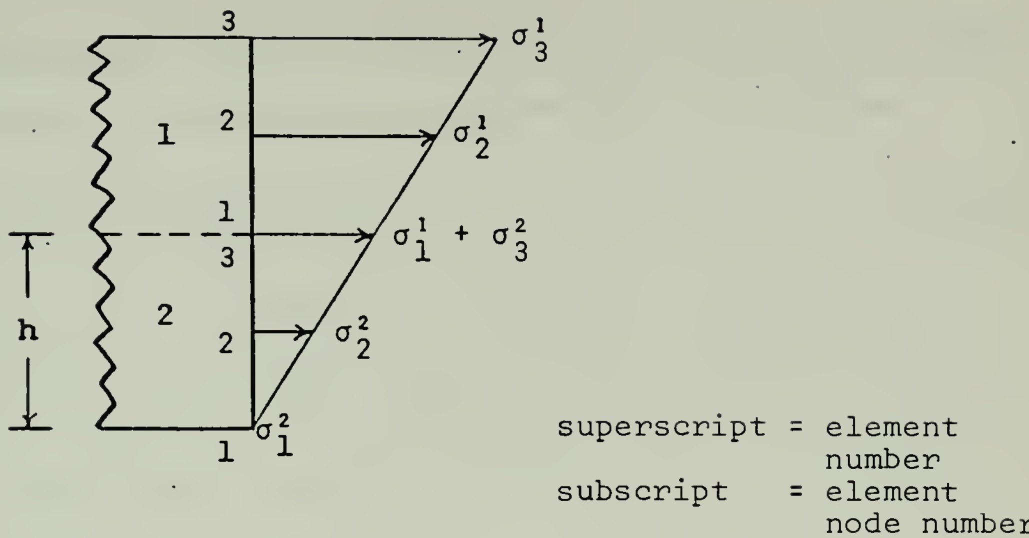


Figure 26. Global bending stress distribution.

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix} = \begin{bmatrix} f_3^1 \\ f_2^1 \\ f_1^1 + f_3^2 \\ f_2^2 \\ f_1^2 \end{bmatrix} = \frac{h\sigma}{6} \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \\ 0 \end{bmatrix} . \tag{14}$$

3. Axisymmetric Stress Analysis of the Axially Loaded Bar

In this case, the region associated with an element is the sector of an annulus with a one radian central angle. Figure 27 illustrates the coordinates used with the axisymmetric element and the node numbering used in development below.

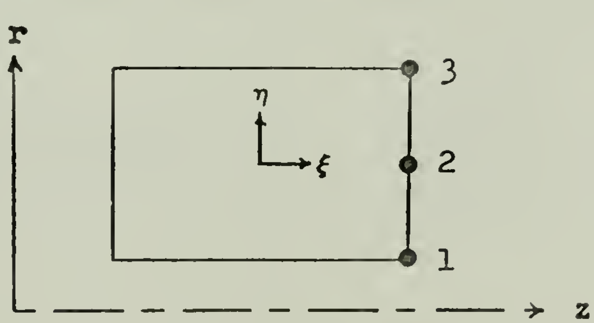


Figure 27. Axisymmetric element.

Beginning with equation (10), the equivalent nodal forces for a uniform external stress are developed as follows:

$$[f_i]^e = \int \sigma \cdot [N_i] d(\text{area}) \tag{10}$$

$$d(\text{area}) = r\ (1\ \text{radian})\ dr = r\ dr$$

$$r = r_1 N_1 + r_2 N_2 + r_3 N_3$$

$$dr = (r_1 N_1' + r_2 N_2' + r_3 N_3') d\eta$$

where $N_i' = \partial N_i / \partial \eta$. Then,

$$dr = \frac{r_3 - r_1}{2}\, d\eta \ .$$

Therefore,

$$d(\text{area}) = \frac{r_3 - r_1}{2} (r_1 N_1 + r_2 N_2 + r_3 N_3) d\eta \ .$$

Substituting back into equation (10),

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \sigma \cdot \frac{r_3 - r_1}{2} \begin{bmatrix} \int (N_1)^2 d\eta & \int N_1 N_2 d\eta & \int N_1 N_3 d\eta \\ & \int (N_2)^2 d\eta & \int N_2 N_3 d\eta \\ \text{sym.} & & \int (N_3)^2 d\eta \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

integrating the shape functions with $\xi = 1$, leaves

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \sigma \cdot \frac{r_3 - r_1}{30} \begin{bmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \ . \tag{15}$$

APPENDIX C: DEVELOPMENT OF THE AXISYMMETRIC STRESS CAPABILITY

For an axisymmetrically loaded bar, displacements are limited to the axial and radial directions. In this study these displacements were identified with the letters u and v, respectively. Cylindrical coordinates are applicable and are defined in Fig. 28. It can be seen that radial displacements introduce a circumferential component of strain

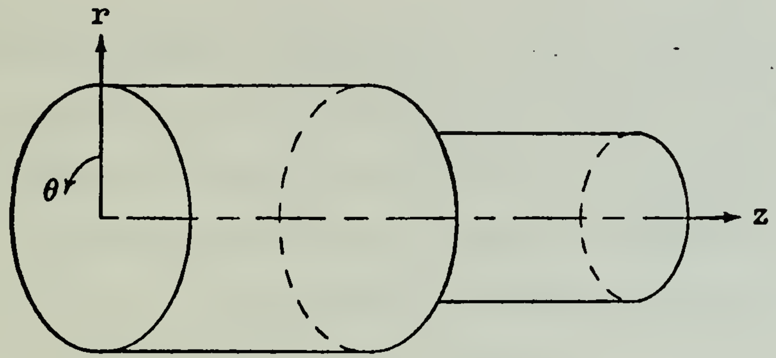


Figure 28. Cylindrical coordinates.

as well as a radial component. For problems of this type, the general strain-displacement relationships, as developed in chapter five of Ref. 4, are

$$[\varepsilon] = \begin{bmatrix} \varepsilon_z \\ \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{rz} \end{bmatrix} = \begin{bmatrix} \partial u/\partial z \\ \partial v/\partial r \\ v/r \\ \partial v/\partial z + \partial u/\partial r \end{bmatrix} . \tag{16}$$

The elasticity matrix which links the strains to the stresses is derived from the following stress-strain relationships:

$$\varepsilon_z = \frac{1}{E}\,[\sigma_z - \nu(\sigma_r + \sigma_\theta)]$$

$$\varepsilon_r = \frac{1}{E}\,[\sigma_r - \nu(\sigma_z + \sigma_\theta)]$$

$$\varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu(\sigma_r + \sigma_z)]$$

$$\gamma_{rz} = \tau_{rz}/G$$

The resulting elasticity matrix [D] is

$$[D] = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \nu/(1-\nu) & \nu/(1-\nu) & 0 \\ & 1 & \nu/(1-\nu) & 0 \\ \text{sym.} & & 1 & 0 \\ & & & (1-2\nu)/2(1-\nu) \end{bmatrix} . \quad (17)$$

The finite element formulation of the element stiffness matrix is carried out as in the plane stress problem, except the integration over the volume of the element must take into consideration the fact that the element thickness varies with radius. The element stiffness matrix [k] can therefore be expressed by the equation,

$$[k]^e = \int [B]^T[D][B] r \, dr \, dz \quad (18)$$

where [B] is the matrix derived from the strain-displacement relationships and defined such that

$$[\varepsilon] = [B][\alpha]^e$$

where $[\alpha]^e$ is the element displacement vector.

The primary changes to the PLISOP program were to incorporate the new [B], [D], and [k] matrices, as defined above. The element shape functions also had to be added, since only their derivatives were included PLISOP.

For further detail on axisymmetric stress analysis by the finite element method, the reader is referred to chapter five of Ref. 4.

APPENDIX D: NOTES ON AXISYMMETRIC STRESS ANALYSIS CAPABILITY OF PLISOP

1. z and r denote respectively the axial and radial coordinates and u and v the corresponding displacements. (In PLISOP, $z \equiv x$ and $r \equiv y$.)

2. The volume of material associated with an element has a thickness equal to the arc subtended by a one radian angle vice the body of revolution usually discussed in most texts. It turns out the only difference is a factor of 2π.

3. Since the axisymmetric analysis involves division by r, there cannot be any nodal points with coordinate $r=0$. For nodal points along the centerline, substituting $r=1 \times 10^{-6}$ suffices to prevent the solution from "blowing up," although the accuracy of stress values near these nodal points is adversely affected.

APPENDIX E: COMPUTER PROGRAM

```
C                                                                         CAN00020
C    ******************************************************************CAN00030
C    THIS PROGRAM PERFORMS A PLANE STRESS,PLANE STRAIN,OR AN AXISYM-     DMD00040
C    METRIC STRESS ANALYSIS BY THE FINITE ELEMENT METHOD. NUMERICALLY    DMD00050
C    INTEGRATED ISOPARAMETRIC ELEMENTS ARE USED THROUGHOUT               DMD00060
C    PROFESSOR GILLES CANTIN,  MARCH 1972                                CAN00070
C    (AXISYMMETRIC STRESS CAPABILITY ADDED BY LCDR D.M.DOYLE DEC.,1974)  DMD00075
C    ******************************************************************CAN00080
C                                                                         CAN00090
C   INPUT CARDS NEEDED                                                    CAN00100
C                                                                         CAN00110
C TITLE                      (20A4)          ONE CARD PER PROBLEM         CAN00120
C                                                                         CAN00130
C  PROBLEM PARAMETERS       (10I5)           ONE CARD PER PROBLEM         CAN00140
C   COL.    VARIABLE                                                      CAN00150
C    1-5        NEL   NUMBER OF ELEMENTS (MAX. IS 182 ELEMENTS, FOR CUBIC CAN00160
C                     ELEM. (6X6), FOR QUAD. (7X8) , FOR LIN. (13X14) )   CAN00170
C    6-10       NJT   NUMBER OF JOINTS (MAX. IS 217)                      CAN00180
C   11-15       NMAT   NUMBER OF MATERIALS (MAX. IS 10)                   CAN00190
C   16-20       NCLOAD  NUMBER OF CONCENTRATED LOADS                      CAN00200
C   21-25       NPBC   NUMBER OF JOINTS WITH BOUNDARY CONDITIONS          CAN00210
C   26-30       NSTRES  ZERO FOR PLANE STRESS PROBLEMS, ONE FOR PLANE     DMD00220
C                      STRAIN, TWO FOR AXISYMMETRIC STRESS                DMD00225
C   31-35       NGP   NUMBER OF GAUSS POINTS DESIRED IN THE INTEGRATION   CAN00230
C   36-40       NTLD  1  IF A THERMAL LOAD IS PRESENT  0  OTHERWISE       CAN00240
C   41-45       NTBY  1  IF A BODY FORCE LOAD IS PRESENT  0 OTHERWISE     CAN00250
C   46-50       NTIN  1  IF AN INITIAL STRETCH IS TO BE CONSIDERED        CAN00260
C               IF NTLD, NTBY OR NTIN ARE DIFFERENT THAN 0, THEN NJT      CAN00270
C               CARDS WILL BE NEEDED BELOW FOR THE CONSISTENT LOAD VECTOR CAN00280
C                                                                         CAN00340
C  JOINT CARDS            (I10,4G15.8,1I5)   NJT CARDS PER PROBLEM        CAN00350
C   COL.    VARIABLE                                                      CAN00360
C    1-10       JOINT NUMBER                                              CAN00370
C   11-25       X   COORDINATE OF THE JOINT                               CAN00380
C   26-40       Y   COORDINATE OF THE JOINT                               CAN00390
C   41-55       X   COMPONENT OF THE LOAD AT THE JOINT                    CAN00400
C   56-70       Y   COMPONENT OF THE LOAD AT THE JOINT                    CAN00410
C   71-75       IND  IF IND IS NOT  0  THE COORDINATE SYSTEM IS POLAR,    CAN00420
C               X  IS THE RADIUS VECTOR, Y  IS THE POLAR ANGLE IN DEGREES CAN00430
C               FOR THE LOAD, X  IS A RADIAL LOAD AND Y  IS A TANGENT LOADCAN00440
C                                                                         CAN00450
C  MATERIAL PROPERTIES       (1I10,4F10.2)  NMAT  CARDS PER PROBLEM       CAN00460
C   COL.    VARIABLE                                                      CAN00470
C    1-10       MATERIAL NUMBER                                           CAN00480
C   11-20       YOUNG'S MODULUS                                           CAN00490
C   21-30       POISSON'S RATIO                                           CAN00500
C   31-40       COEFFICIENT OF THERMAL EXPANSION                          CAN00510
C   41-50       THICKNESS OF THE ELEMENT                                  CAN00520
```

```
C                                                                       CAN00530
C     ELEMENT CARDS              (15I4)          NEL   CARDS PER PROBLEM CAN00540
C      COL.    VARIABLE                                                 CAN00550
C       1-4        ELEMENT IDENTIFICATION NUMBER                        CAN00560
C       5-52       ELEMENT CONNECTIVITY (COUNTERCLOCKWISE)              CAN00570
C      53-56       KIND  1   FOR FOUR NODAL POINT ELEMENT               CAN00580
C                        2   FOR EIGHT NODAL POINT ELEMENT              CAN00590
C                        3   FOR TWELVE NODAL POINT ELEMENT             CAN00600
C      57-60       TYPE  THIS IS THE MATERIAL IDENTIFICATION NUMBER     CAN00610
C                                                                       CAN00640
C     BOUNDARY CONDITION CARDS  (2I5)            NPBC  CARDS PER PROBLEM CAN00650
C      COL.    VARIABLE                                                 CAN00660
C       1-5  JOINT NUMBER                                               CAN00670
C       6-10 CODE   1 FOR A ROLLER ON AN X AXIS, DISP. IN Y DIRECT. IS ZERO CAN00680
C                   2 FOR A ROLLER ON AN Y AXIS, DISP. IN X DIRECT. IS ZERO CAN00690
C                   3 FOR A FIXED JOINT, BOTH DISP. COMPONENTS ARE ZERO CAN00700
C                   4 FOR A ROLLER ON AN X AXIS, THE DISP. IN THE Y DIRECT. CAN00710
C                     IS SPECIFIED ON THE JOINT CARD IN COL. 56-70      CAN00720
C                   5 FOR A ROLLER ON A Y AXIS, THE DISP. IN THE X DIRECT. CAN00730
C                     IS SPECIFIED ON THE JOINT CARD IN COL. 41-55      CAN00740
C                   6 FIXED JOINT WITH DISP. SPECIFIED IN BOTH X AND Y  CAN00750
C                     DIRECT. ON THE JOINT CARD IN COL.41-55 AND COL.56-70 CAN00760
C                   7 FOR A ROLLER ON AN AXIS INCLINED ALPHA DEGREES    CAN00770
C                     WITH RESPECT TO THE X AXIS. THE JOINT CARD MAY    CAN00780
C                     CONTAIN A TANGENTIAL LOAD IN COL. 41-55 AND THE   CAN00790
C                     ANGLE ALPHA DEGREES IN COL. 56-70                 CAN00800
C                                                                       CAN00810
C     CONSISTENT LOAD VECTOR (I10,2G25.16)     NJT CARDS PER PROBLEM    CAN00820
C      NO CARDS NEEDED IF THERE ARE NO DISTRIBUTED LOADS                CAN00830
C      COL.    VARIABLE                                                 CAN00840
C       1-10 JOINT NUMBER                                               CAN00850
C      11-35 X COMPONENT OF LOAD                                        CAN00860
C      36-70 Y COMPONENT OF LOAD                                        CAN00870
C                                                                       CAN00880
C     AS MANY PROBLEMS AS ONE DESIRES MAY BE STACKED, BUT THE LAST CARD OF CAN00890
C     A RUN MUST CONTAIN THE WORD    STOP    IN  THE FIRST FOUR COLUMNS CAN00900
C                                                                       CAN00910
C      PROFESSOR GILLES CANTIN AT THE NAVAL POSTGRADUATE SCHOOL  1972   CAN00920
C                                                                       CAN00930
C        FOR AXISYMMETRIC STRESS ANALYSIS:                              DMD00940
C            -COORDINATES (Z,R) ARE EQUIVALENT TO (X,Y)                 DMD00950
C            - R CAN NEVER EQUAL ZERO,USE R=.000001 FOR CENTERLINE NODES DMD00960
C            -THE REGION ASSOCIATED WITH AN ELEMENT IS THE SECTOR OF AN DMD00970
C             ANNULUS WITH A ONE RADIAN CENTRAL ANGLE                   DMD00980
C
C           MAIN  PROGRAM
C
        IMPLICIT REAL*8(A-H,O-Z)                                        CAN00290
```

```
      COMMON/SOL/BGK(434,74),ALOAD(434),ABGN                            CAN00300
      COMMON/INT/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,14),NBC(217,2),NSTRESCAN00310
     1,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN                                CAN00320
      COMMON/MDIM/MEL,MJT,MMT,MBD
      COMMON/FLPL/COARD(217,2),CLOAD(217,2),ELCON(10,4),TITLE(10)       CAN00330
      DIMENSION REACT(218,2),NCODE(217)                                 CAN00620
      EQUIVALENCE (BGK(1,1),REACT(1,1))                                 CAN00630
      MEL=182
      MJT=217
      MMT=10
      MBD=74
      PI=3.14159265359D0                                                CAN00940
      ZRO=0.0D0                                                         CAN00950
   10 NBAND=0                                                           CAN00960
      DO 20 I=1,MJT
   20 NCODE(I)=0                                                        CAN00980
      CALL INPUT                                                        CAN00990
      NEQ=2*NJT                                                         CAN01000
      NPEL=NCON(1,13)*4                                                 CAN01010
      DO 300 I=1,NEL                                                    CAN01020
      DO 100 J=1,NPEL                                                   CAN01030
  100 LM(J)=NCON(I,J)                                                   CAN01040
      NPELM=NPEL-1                                                      CAN01050
      DO 200 K=1,NPELM                                                  CAN01060
      JK=K+1                                                            CAN01070
      DO 200 L=JK,NPEL                                                  CAN01080
  200 NBAND=MAX0(NBAND,IABS(LM(K)-LM(L)))                               CAN01090
  300 CONTINUE                                                          CAN01100
      NBAND=(NBAND+1)*2                                                 CAN01110
      IF(NBAND.GT.74) GO TO 6000                                        CAN01120
      DO 350 I=1,NEQ                                                    CAN01130
      ALOAD(I)=ZRO                                                      CAN01140
      DO 350 J=1,NBAND                                                  CAN01150
  350 BGK(I,J)=ZRO                                                      CAN01160
      CALL MERGE(NPEL,NEQ)                                              CAN01170
      CALL BCOND(NBAND)                                                 CAN01180
      CALL LDLT(NEQ,NBAND)                                              CAN01190
      DO 500 I=1,NPBC                                                   CAN01200
      IJT=NBC(I,1)                                                      CAN01210
  500 NCODE(IJT)=NBC(I,2)                                               CAN01220
      DO 600 I=1,NJT                                                    CAN01230
      II=I*2-1                                                          CAN01240
      KODE=NCODE(I)                                                     CAN01250
      IF(KODE.EQ.7) GO TO 560                                           CAN01260
      IF((KODE.EQ.5).OR.(KODE.EQ.6)) GO TO 550                          CAN01270
      ALOAD(II)=CLOAD(I,1)+ALOAD(II)                                    CAN01280
  550 II=II+1                                                           CAN01290
      IF((KODE.EQ.4).OR.(KODE.EQ.6)) GO TO 600                          CAN01300
```

```
      ALOAD(II)=CLOAD(I,2)+ALOAD(II)                                    CAN01310
      GO TO 600                                                         CAN01320
  560 ALF=CLOAD(I,2)*PI/180.0D0                                         CAN01330
      COSA=DCOS(ALF)                                                    CAN01340
      SINA=DSIN(ALF)                                                    CAN01350
      IIP=II+1                                                          CAN01360
      A1=ALOAD(II)*COSA+ALOAD(IIP)*SINA+CLOAD(I,1)                      CAN01370
      A2=ALOAD(IIP)*COSA-ALOAD(II)*SINA                                 CAN01380
      ALOAD(II)=A1                                                      CAN01390
      ALOAD(IIP)=A2                                                     CAN01400
  600 CONTINUE                                                          CAN01410
      CALL SOLV(NEQ,NBAND)                                              CAN01420
      DO 630 I=1,NPBC                                                   CAN01430
      DO 630 J=1,2                                                      CAN01440
  630 REACT(I,J)=ZRO                                                    CAN01450
      CALL DISPL                                                        CAN01460
      WRITE(6,1000)                                                     CAN01470
      DO 700 I=1,NJT                                                    CAN01480
      II=I*2-1                                                          CAN01490
      III=II+1                                                          CAN01500
  700 WRITE(6,2000) I,ALOAD(II),ALOAD(III)                              CAN01510
 2000 FORMAT(2X,I3,2G15.4)                                              CAN01520
 1000 FORMAT(//,' DISPLACEMENTS',//,3X,' JOINT',1X,'  U',12X,'  V',/)   CAN01530
      IF(NSTRES.EQ.2) GO TO 1250                                        DMD01535
      WRITE(6,3000)                                                     CAN01540
      GO TO 750                                                         DMD01541
 1250 WRITE(6,1350)                                                     DMD01545
  750 DO 800 I=1,NPBC                                                   CAN01550
  800 WRITE(6,2000) NBC(I,1),REACT(I,1),REACT(I,2)                      CAN01555
 3000 FORMAT(//,'      REACTIONS',//,3X,' JOINT',1X,' RX',12X,' RY',/)  CAN01560
 1350 FORMAT(//,'      REACTIONS',//,3X,' JOINT',1X,' RZ',12X,' RR',/)  DMD01565
      WRITE(6,4000)                                                     CAN01580
 4000 FORMAT(/,' EQUILIBRIUM CHECK ',/)                                 CAN01590
      MJP=MJT+1
      WRITE(6,2500) REACT(MJP,1),REACT(MJP,2)
 2500 FORMAT(5X,2G15.4)                                                 CAN01610
      CALL STRESS(NPEL,NSTRES)
      GO TO 10                                                          CAN01630
 6000 WRITE(6,6500) MBD,NBAND
 6500 FORMAT(5X,' MAX. BAND (',I13,') EXCEEDED, NBAND = ',I14)
      STOP                                                              CAN01660
      END                                                               CAN01670
      SUBROUTINE INPUT                                                  CAN01680
      IMPLICIT REAL*8(A-H,O-Z)                                          CAN01690
      COMMON/INT/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,14),NBC(217,2),NSTRESCAN01700
     1,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN                                CAN01710
      COMMON/MDIM/MEL,MJT,MMT,MBD
      COMMON/FLPL/COAFD(217,2),CLOAD(217,2),ELCON(10,4),TITLE(10)       CAN01720
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```
      DATA CHK /'STOP    '/                                             CAN01730
      ZRO=0.0D0                                                         CAN01740
      PI=3.14159265359D0                                                CAN01750
      READ (5,1000) TITLE                                               CAN01760
 1000 FORMAT (10A8)                                                     CAN01770
      IF (TITLE(1).EQ.CHK) STOP                                         CAN01780
      DO 1050 I=1,MJT
      DO 1050 J=1,2                                                     CAN01800
      NBC(I,J)=0                                                        CAN01810
      COARD(I,J)=ZRO                                                    CAN01820
 1050 CLOAD(I,J)=ZRO                                                    CAN01830
      DO 1060 I=1,MMT
      DO 1060 J=1,4                                                     CAN01850
 1060 ELCON(I,J)=ZRO                                                    CAN01860
      DO 1070 I=1,MEL
      DO 1070 J=1,14                                                    CAN01880
 1070 NCON(I,J)=0                                                       CAN01890
      WRITE(6,1000) TITLE                                               CAN01900
C                                                                       CAN01910
C     READ STRUCTURE PARAMETERS                                         CAN01920
C                                                                       CAN01930
 1100 FORMAT(10I5)                                                      CAN01940
      READ(5,1100) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NTLD,NTBY,NTIN   CAN01950
      IF(NEL.GT.MEL) GO TO 4300
      IF(NJT.GT.MJT) GO TO 4400
      IF(NMAT.GT.MMT) GO TO 4500
      IF(NCLOAD.GT.MJT) GO TO 4600
      IF(NPBC.GT.MJT) GO TO 4700
      WRITE (6,1200) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP                CAN02010
 1200 FORMAT (//,' NUMBER OF ELEMENTS=',I5,//,' TOTAL NUMBER OF JOINTS='CAN02020
     1,I5,//,' NUMBER OF MATERIALS=',I5,//,' NUMBER OF CONCENTRATED LOADCAN02030
     2S=',I5,//,' NUMBER OF JOINTS WITH BOUNDARY CONDITIONS=',I5,//,' THCAN02040
     3E VALUE OF NSTRES=',I5,//,' NUMBER OF GAUSS POINTS USED ',I5)     CAN02050
C                                                                       CAN02060
C     READ COORDINATES OF JOINTS AND CONCENTRATED LOADS                 CAN02070
C                                                                       CAN02080
      IF(NSTRES.EQ.2) GO TO 1250                                        DMD02085
      WRITE (6,1300)                                                    CAN02090
      GO TO 1260                                                        DMD02091
 1250 WRITE(6,1350)                                                     DMD02095
 1260 DO 1400 I=1,NJT                                                   CAN02120
      READ(5,1500) IJT,COARD(IJT,1),COARD(IJT,2),CLOAD(IJT,1),          CAN02130
     1CLOAD(IJT,2),IND                                                  CAN02140
      IF(IND.EQ.0) GO TO 1400                                           CAN02150
      ALF=COARD(IJT,2)*PI/180.0D0                                       CAN02160
      COSA=DCOS(ALF)                                                    CAN02170
      SINA=DSIN(ALF)                                                    CAN02180
      XC=COARD(IJT,1)*COSA                                              CAN02190
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      YC=COARD(IJT,1)*SINA                                              CAN02200
 1400 CONTINUE                                                          CAN02230
 1300 FORMAT (///,' JOINT NUMBER',5X,'X COORDINATE',5X,'Y COORDINATE',7XCAN02240
     1,'X LOAD',9X,' Y LOAD'//)                                         CAN02241
 1350 FORMAT (///,' JOINT NUMBER',5X,'Z COORDINATE',5X,'R COORDINATE',7XDMD02245
     1,'Z LOAD',9X,' R LOAD'//)                                         DMD02246
 1500 FORMAT(I10,4G15.8,I5)
      DO 1600 I=1,NJT                                                   CAN02250
 1600 WRITE (6,1700) I,COARD(I,1),COARD(I,2),CLOAD(I,1),CLOAD(I,2)      CAN02260
 1700 FORMAT      (5X,I3,10X,G14.5,3X,G14.5,5X,G14.5,3X,G14.5)          CAN02270
C                                                                       CAN02280
C     READ MATERIAL PROPERTIES                                          CAN02290
C                                                                       CAN02300
      DO 1800 I=1,NMAT                                                  CAN02310
 1800 READ(5,1900) IMAT,(ELCON(IMAT,J),J=1,4)                           CAN02320
 1900 FORMAT(1I10,4F10.2)                                               CAN02330
      WRITE (6,2000)                                                    CAN02340
 2000 FORMAT(///,' MATERIAL PROPERTIES',//,3X,' MAT.NO. YOUNG,S MOD.  POCAN02350
     1ISSON,S RAT.      ALPHA         THICKNESS',//)                    CAN02360
      DO 2100 I=1,NMAT                                                  CAN02370
 2100 WRITE(6,2200) I,(ELCON(I,J),J=1,4)                                CAN02380
 2200 FORMAT(6X,1I3,1X,4G15.6)                                          CAN02390
C                                                                       CAN02400
C     READ CONNECTIVITY MATRIX                                          CAN02410
C                                                                       CAN02420
      WRITE (6,2300)                                                    CAN02430
 2300 FORMAT (///,' CONNECTIVITY MATRIX',//,'    EL NO',59X,' KIND',' TYPCAN02440
     1E'//)                                                             CAN02450
      DO 2400 I=1,NEL                                                   CAN02460
 2400 READ (5,2500) IJT,(NCON(IJT,J),J=1,14)                            CAN02470
 2500 FORMAT (15I4)                                                     CAN02480
      DO 2600 I=1,NEL                                                   CAN02490
 2600 WRITE (6,2700) I,(NCON(I,J),J=1,14)                               CAN02500
 2700 FORMAT( 2X,I3,14I5)                                               CAN02510
C                                                                       CAN02520
C     READ BOUNDARY CONDITION CODES                                     CAN02530
C                                                                       CAN02540
      WRITE (6,2800)                                                    CAN02550
 2800 FORMAT (///,' BOUNDARY CONDITION CODE',//,5X,'JNT NO',6X,'CODE'//)CAN02560
      DO 2900 I=1,NPBC                                                  CAN02570
      READ (5,3000) (NBC(I,J),J=1,2)                                    CAN02580
 3000 FORMAT (2I5)                                                      CAN02590
 2900 WRITE (6,3100) (NBC(I,J),J=1,2)                                   CAN02600
 3100 FORMAT( 5X,I5,I10)                                                CAN02610
      NPBCM=NPBC-1                                                      CAN02620
      NPBCT=(NPBC/2)+1                                                  CAN02630
      DO 4000 K=1,NPBCT                                                 CAN02640
      DO 4000 I=1,NPBCM                                                 CAN02650
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      IF(NBC(I,2).EQ.7) GO TO 3600                                      CANO2660
      GO TO 4000                                                        CANO2670
 3600 N1=NBC(I,1)                                                       CANO2680
      DO 3800 J=I,NPBCM                                                 CANO2690
      NBC(J,1)=NBC(J+1,1)                                               CANO2700
 3800 NBC(J,2)=NBC(J+1,2)                                               CANO2710
      NBC(NPBC,1)=N1                                                    CANO2720
      NBC(NPBC,2)=7                                                     CANO2730
 4000 CONTINUE                                                          CANO2740
C                                                                       CANO2750
C     READ LOAD VECTOR  FROM SERVICE PROGRAM                            CANO2760
C                                                                       CANO2770
      IF((NTLD+NTBY+NTIN).EQ.0) RETURN                                  CANO2780
      DO 4200 I=1,NJT                                                   CANO2790
      READ(5,4100) IJT,CLX,CLY                                          CANO2800
 4100 FORMAT(5X,1I5,2G25.16)                                            CANO2810
      CLOAD(IJT,1)=CLOAD(IJT,1)+CLX                                     CANO2820
      CLOAD(IJT,2)=CLOAD(IJT,2)+CLY                                     CANO2830
 4200 CONTINUE                                                          CANO2840
      RETURN                                                            CANO2850
 4300 WRITE(6,4350) MEL,NEL
 4350 FORMAT(5X,' MAX. NUMBER (',1I3,') OF ELEMENTS EXCEEDED, NEL=',1I4)
      STOP                                                              CANO2880
 4400 WRITE(6,4450) MJT,NJT
 4450 FORMAT(5X,' MAX. NUMB. (',1I3,') OF JOINTS EXCEEDED, NJT =',1I4)
      STOP                                                              CANO2910
 4500 WRITE(6,4550) MMT,NMAT
 4550 FORMAT(5X,' MAX. NUMB. (',1I2,') OF MATERIAL EXCEEDED, NMAT=',1I4)
      STOP                                                              CANO2940
 4600 WRITE(6,4650) MJT,NCLOAD
 4650 FORMAT(5X,' MAX. NUMB. (',1I3,') OF C. LOADS EXC., NCLOAD =',1I4)
      STOP                                                              CANO2970
 4700 WRITE(6,4750) MJT,NPBC
 4750 FORMAT(5X,' MAX. N. (',1I3,') OF JOINTS WITH BOUND. COND. EXCDED,
     1 NPBC =',1I4)                                                     CANO3000
      STOP                                                              CANO3010
      END                                                               CANO3020
      SUBROUTINE MERGE(NPEL,NEQ)                                        CANO3030
      IMPLICIT REAL*8(A-H,O-Z)                                          CANO3040
      DIMENSION          STK(24,24),AK(24,24),B(4,24)                   CANO3050
      COMMON/SOL/BCK(434,74),ALOAD(434),ABGN                            CANO3060
      COMMON/INT/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,14),NBC(217,2),NSTRESCANO3070
     1,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN                                CANO3080
      COMMON/FLPL/COARD(217,2),CLOAD(217,2),ELCON(10,4),TITLE(10)       CANO3090
      COMMON COORD(12,2),ELAST(4,4),SS(48,24),SN(48,24)                 DMDO3100
      REWIND 10                                                         CANO3110
      ZRO=0.0D0                                                         CANO3120
      DO 100 I=1,48                                                     DMDO3130
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      DO 100 J=1,24                                                     CAN03140
      SS(I,J)=ZRO                                                       CAN03150
  100 SN(I,J)=ZRO                                                       CAN03160
      DO 200 IK=1,NEL                                                   CAN03170
      DO 110 JJ=1,NPEL                                                  DMD03180
      LJT(JJ)=NCON(IK,JJ)                                               DMD03190
  110 LM(JJ)=(NCON(IK,JJ)-1)*2                                          DMD03200
      NS=4*NPEL                                                         DMD03210
      DO 120 II=1,NPEL                                                  CAN03220
      I2=LJT(II)                                                        CAN03230
      DO 120 J1=1,2                                                     CAN03240
  120 COORD(II,J1)=COARD(I2,J1)                                         CAN03250
      MATN=NCON(IK,14)                                                  CAN03260
      N=NCON(IK,13)*8                                                   CAN03270
      YM=ELCON(MATN,1)                                                  CAN03280
      PRTIO=ELCON(MATN,2)                                               CAN03290
      TK=ELCON(MATN,4)                                                  CAN03300
      IF(NSTRES.GE.1) TK=1.0D0                                          DMD03310
      CALL ELASM(ELAST,YM,PRTIO,NSTRES)                                 CAN03320
      CALL QUAD5(STK,AK,B,NS,N,NGP,NSTRES)                              CAN03330
      WRITE(10) SS,SN                                                   CAN03340
      IF(TK.EQ.0.0D0) TK=1.0D0                                          CAN03350
      DO 200 I=1,NPEL                                                   CAN03360
      DO 200 J=1,NPEL                                                   CAN03370
      DO 200 K=1,2                                                      CAN03380
      II=LM(I)+K                                                        CAN03390
      KK=2*(I-1)+K                                                      CAN03400
      DO 200 L=1,2                                                      CAN03410
      JJ=LM(J)+L-II+1                                                   CAN03420
      IF(JJ.LE.0) GO TO 200                                             CAN03430
      LL=2*(J-1)+L                                                      CAN03440
      BGK(II,JJ)=BGK(II,JJ)+STK(KK,LL)*TK                               CAN03450
  200 CONTINUE                                                          CAN03460
      SUM=ZRO                                                           CAN03470
      CDN=ZRO                                                           CAN03480
      DO 300 I=1,NEQ                                                    CAN03490
  300 SUM=SUM+BGK(I,1)                                                  CAN03500
      ABGN=SUM*1.0D20                                                   CAN03510
      RETURN                                                            CAN03520
      END                                                               CAN03530
      SUBROUTINE ELASM(D,E,PR,NN)                                       CAN03540
      IMPLICIT REAL*8(A-H,O-Z)                                          CAN03550
C     THIS SUBROUTINE ASSEMBLYS THE ELASTIC MATRIX FOR PLANE            CAN03560
C     STRAIN (NN=1) OR PLANE STRESS (NN=0) OR AXISYMMETRIC STRESS(NN=2) CAN03570
      DIMENSION D(4,4)                                                  CAN03580
      ON=1.0D0                                                          CAN03590
      TW=2.0D0                                                          CAN03600
      DO 20 I=1,4                                                       CAN03610
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      DO 20 J=1,4                                                       CAN03620
   20 D(I,J)=0.0D0                                                      CAN03630
      IF(NN-1) 11,10,12                                                 DMD03640
C     FORM ELASTIC MATRIX FOR PLANE STRAIN                              CAN03650
   10 ER=E/((ON+PR)*(ON-TW*PR))                                         CAN03660
      D(1,1)=ER*(ON-PR)                                                 CAN03670
      D(1,2)=ER*PR                                                      CAN03680
      D(2,1)=D(1,2)                                                     CAN03690
      D(2,2)=D(1,1)                                                     CAN03700
      D(3,3)=ER*(.5D0-PR)                                               CAN03710
      RETURN                                                            CAN03720
C     FORM ELASTIC MATRIX FOR PLANE STRESS                              CAN03730
   11 ER=E/(ON-PR**2)                                                   CAN03740
      D(1,1)=ER                                                         CAN03750
      D(1,2)=ER*PR                                                      CAN03760
      D(2,1)=D(1,2)                                                     CAN03770
      D(2,2)=D(1,1)                                                     CAN03780
      D(3,3)=ER*(1.-PR)/TW                                              CAN03790
      RETURN                                                            CAN03800
C     FORM ELASTIC MATRIX FOR AXISYMMETRIC STRESS                       DMD 3801
   12 ER=(E*(ON-PR))/((ON+PR)*(ON-2.0D0*PR))                            DMD03802
      D(1,1)=ER                                                         DMD03803
      D(1,2)=ER*(PR/(ON-PR))                                            DMD03804
      D(1,3)=ER*(PR/(ON-PR))                                            DMD03805
      D(2,1)=D(1,2)                                                     DMD03806
      D(2,2)=ER                                                         DMD03807
      D(2,3)=D(1,2)                                                     DMD03808
      D(3,1)=D(1,3)                                                     DMD03809
      D(3,2)=D(2,3)                                                     DMD03810
      D(3,3)=ER                                                         DMD03811
      D(4,4)=ER*((ON-2.0D0*PR)/(2.0D0*(ON-PR)))                         DMD03812
      RETURN                                                            DMD03813
      END                                                               DMD03814
C                                                                       CAN03840
C    QUAD5 IS A NUMERICAL INTEGRATION SUBROUTINE THAT FORMS THE STIFFNESSCAN03850
C    MATRIX FOR ANY OF THE THREE QUADRILATERAL ELEMENTS IN THIS FAMILY.  CAN03860
C    GAUSSIAN QUADRATURE WITH A MAXIMUM OF FIVE ORDINATES IS USED.       CAN03870
C                                                                       CAN03880
C    THE SUBROUTINE IS CALLED WITH THE FOLLOWING STATEMENT:             CAN03890
C    CALL QUAD5(STK,AK,SS,SN,NS,N,NGP)                                  CAN03900
C    THE CALLING PROGRAM MUST CONTAIN THE FOLLOWING COMMON STATEMENT:   CAN03910
C    COMMON COORD(12,2),ELAST(4,4)                                      DMD03920
C           STK  IS AN ARRAY DIMENSIONED NXN THAT WILL CONTAIN THE STIFFNESCAN03930
C                MATRIX UPON RETURN TO THE CALLING PROGRAM              CAN03940
C        SS AND SN ARE THE STRESS AND STRAIN VERSUS NODAL POINT DISPLACECAN03950
C                TRANSFORMATION MATRIX, BOTH ARE DIMENSIONED NSXN WHERE CAN03960
C                NS IS 3*NPT AND N IS 2*NPT                             CAN03970
C                      (NS IS 4*NPT FOR AXISYMMETRIC PROBLEMS)          DMD03975
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C        AK    IS AN ARRAY DIMENSIONED NXN REQUIRED IN FORMK                 CAN03980
C        B     IS AN ARRAY DIMENSIONED 2XN REQUIRED IN FORMK                 CAN03990
C                                                                             CAN04000
C        NGP IS THE NUMBER OF GAUSS POINTS UED IN THE INTEGRATION            CAN04010
C                                                                             CAN04020
C        COORD(12,2) IS AN ARRAY CONTAINING THE COORDINATES OF ALL           CAN04030
C                    THE NODAL POINTS OF THE ELEMENT TO BE CONSTRUCTED       CAN04040
C        ELAST(4,4)  IS AN ARRAY CONTAINING ALL THE ELASTIC CONSTANTS        DMD04050
C                    OF THE ELEMENT, THE ORDER OF THE STRESS COMPONENTS      CAN04060
C                    IS: SIGMAX,SIGMAY,TAUXY OR SIGMAR,SIGMATHETA,           DMD04070
C                        SIGMAZ,SIGMARZ FOR AXISYMMETRIC STRESS              DMD04075
C                                                                             CAN04080
C                                                                             CAN04130
C              ETA                   ETA                    ETA              CAN04140
C               +                     +                      +               CAN04150
C         U2----+-----U1       U3----U2----U1        U4--U3+-U2--U1          CAN04160
C         |     +     |        |     +     |         |      +     |          CAN04170
C         |     +     |        |     +     |         |      +     |          CAN04180
C         |     +     |        |     +     |         |      +     |          CAN04190
C         |     +     |        |     +     |         U5     +     U12        CAN04200
C         |     +     |        |     +     |         |      +     |          CAN04210
C         |     ++++++++XI     U4    ++++++U8++XI    |      +++++++++XI      CAN04220
C         |           |        |           |         |            |          CAN04230
C         |           |        |           |         U6           U11        CAN04240
C         |           |        |           |         |            |          CAN04250
C         |           |        |           |         |            |          CAN04260
C         |           |        |           |         |            |          CAN04270
C         U3----------U4       U5----U6----U7        U7--U8--U9--U10         CAN04280
C                                                                             CAN04290
C     CODED BY PROFESSOR GILLES CANTIN AT THE NAVAL POSTGRADUATE SCHOOL      CAN04300
C                                                                             CAN04310
      SUBROUTINE QUAD5(STK,AK,B,NS,N,NGP,NN)                                 CAN03820
      IMPLICIT REAL*8(A-H,O-Z)                                               CAN03830
      DIMENSION AK(24,24),STK(24,24),B(4,24)                                 CAN04090
      DIMENSION XI(5),AI(5),AIA(5,5)                                         CAN04100
      DIMENSION X2(2),X3(3),X4(4),X5(5),A2(2),A3(3),A4(4),A5(5)              CAN04110
      DIMENSION XYL(4,2),XYQ(8,2),XYC(12,2)                                  CAN04120
      COMMON COORD(12,2),ELAST(4,4),SS(48,24),SN(48,24)                      DMD04120
      DATA X2/.57735026918962D0,-.57735026918962D0/                          CAN04330
      DATA A2/1.0D0,1.0D0/                                                   CAN04340
      DATA X3/.77459666924148 3D0,0.0D0,-.77459666924148 3D0/                CAN04350
      DATA A3/.55555555555556D0,.88888888888889D0,.55555555555556D0/         CAN04360
      DATA X4/.86113631159400D0,.33998104358480D0,-.33998104358480D0,-.8611CAN04370
     13631159400D0/                                                          CAN04380
      DATA A4/.34785484513740D0,.65214515486250D0,.65214515486250D0,.347854CAN04390
     18451374D0/                                                             CAN04400
      DATA X5/.90617984593860D0,.53846931010560D0,0.0D0,-.53846931010560D0,CAN04410
     1-.90617984593360D0/                                                    CAN04420
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      DATA A5/.2369268850561D0,.4786286704993D0,.5688888888888D0,.478628CAN04430
     16704993D0,.2369268850561D0/                                       CAN04440
      DATA XYL/1.0D0,-1.0D0,-1.0D0,1.0D0,1.0D0,1.0D0,-1.0D0,-1.0D0/     CAN04450
      DATA XYQ/1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0,1.0D0,1.0D0,1.0D0,CAN04460
     11.0D0,1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0/                     CAN04470
      DATA XYC/1.0D0,.3333333333333D0,-.3333333333333D0,-1.0D0,-1.0D0,-1CAN04480
     1.0D0,-1.0D0,-.3333333333333D0,.3333333333333D0,1.0D0,1.0D0,1.0D0,1CAN04490
     2.0D0,1.0D0,1.0D0,1.0D0,.3333333333333D0,-.3333333333333D0,-1.0D0,-CAN04500
     31.0D0,-1.0D0,-1.0D0,-.3333333333333D0,.3333333333333D0/           CAN04510
      IF((NGP.EQ.0).OR.(NGP.GT.5)) NGP=3                                CAN04520
      DO 100 I=1,N                                                      CAN04530
      DO 100 J=1,N                                                      CAN04540
  100 STK(I,J)=0.0D0                                                    CAN04550
      JUMP=NGP-1                                                        CAN04560
      DO 140 I=1,NGP                                                    CAN04570
      GO TO (132,133,134,135),JUMP                                      CAN04580
  132 XI(I)=X2(I)                                                       CAN04590
      AI(I)=A2(I)                                                       CAN04600
      GO TO 140                                                         CAN04610
  133 XI(I)=X3(I)                                                       CAN04620
      AI(I)=A3(I)                                                       CAN04630
      GO TO 140                                                         CAN04640
  134 XI(I)=X4(I)                                                       CAN04650
      AI(I)=A4(I)                                                       CAN04660
      GO TO 140                                                         CAN04670
  135 XI(I)=X5(I)                                                       CAN04680
      AI(I)=A5(I)                                                       CAN04690
  140 CONTINUE                                                          CAN04700
      DO 150 I=1,NGP                                                    CAN04710
      DO 150 J=1,NGP                                                    CAN04720
  150 AIA(I,J)=AI(I)*AI(J)                                              CAN04730
      DO 200 I=1,NGP                                                    CAN04740
      X=XI(I)                                                           CAN04750
      DO 200 J=1,NGP                                                    CAN04760
      Y=XI(J)                                                           CAN04770
      CALL FORMK(AK,X,Y,B,N,NN)                                         CAN04780
      AIAIJ=AIA(I,J)                                                    CAN04790
      DO 200 K=1,N                                                      CAN04800
      DO 200 L=1,N                                                      CAN04810
  200 STK(K,L)=STK(K,L)+AIAIJ*AK(K,L)                                   CAN04820
      DO 300 I=1,N                                                      CAN04830
      DO 300 J=I,N                                                      CAN04840
      STK(I,J)=(STK(I,J)+STK(J,I))*0.5D0                                CAN04850
  300 STK(J,I)=STK(I,J)                                                 CAN04860
      NPT=N/2                                                           CAN04870
      IGO=NPT/4                                                         CAN04880
      DO 1000 I=1,NPT                                                   CAN04890
      GO TO (400,500,600),IGO                                           CAN04900
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  400 X=XYL(I,1)                                                          CAN04910
      Y=XYL(I,2)                                                          CAN04920
      GO TO 700                                                           CAN04930
  500 X=XYQ(I,1)                                                          CAN04940
      Y=XYQ(I,2)                                                          CAN04950
      GO TO 700                                                           CAN04960
  600 X=XYC(I,1)                                                          CAN04970
      Y=XYC(I,2)                                                          CAN04980
  700 CALL FORMK(AK,X,Y,B,N,NN)                                           CAN04990
      DO 800 L=1,4                                                        DMD05000
      DO 800 M=1,N                                                        CAN05010
      AK(L,M)=0.0D0                                                       CAN05020
      DO 800 JJ=1,4                                                       DMD05030
  800 AK(L,M)=AK(L,M)+ELAST(L,JJ)*B(JJ,M)                                 DMD05040
      II=4*(I-1)                                                          DMD05050
      DO 900 J=1,4                                                        DMD05060
      IJ=II+J                                                             CAN05070
      DO 900 K=1,N                                                        CAN05080
      SS(IJ,K)=AK(J,K)                                                    CAN05090
  900 SN(IJ,K)=B(J,K)                                                     CAN05100
 1000 CONTINUE                                                            CAN05110
      RETURN                                                              CAN05120
      END                                                                 CAN05130
      SUBROUTINE FORMK(AK,X,Y,B,N,NN)                                     CAN05140
      IMPLICIT REAL*8(A-H,O-Z)                                            CAN05150
C                                                                         CAN05160
C                                                                         CAN05170
C THIS SUBROUTINE FORMS THE STIFFNESS AND THE B MATRIX AS FUNCTIONS OF   CAN05180
C XI AND ETA FOR THE THREE DIFFERENT QUADRILATERAL ELEM. IN THIS FAMILY  CAN05190
C                                                                         CAN05200
C                                                                         CAN05210
      DIMENSION AJ(2,2),AJIN(2,2),DNX(2,12),W1(2,12),B(4,24)              CAN05220
     1,B1(24,4),AK(24,24),S(12)                                           DMD05230
      COMMON COORD(12,2),ELAST(4,4),SS(48,24),SN(48,24)                   DMD05240
C INITIALISE                                                              CAN05250
      ZERO=0.0D0                                                          CAN05260
      DO 10 I=1,4                                                         DMD05270
      DO 10 J=1,N                                                         CAN05280
   10 B(I,J)=ZERO                                                         CAN05290
      ONE=1.0D0                                                           CAN05300
      TWO=2.0D0                                                           CAN05301
      THR=3.0D0                                                           DMD05302
      FOUR=4.0D0                                                          CAN05303
      NINE=9.0D0                                                          DMD05304
      TEN=10.0D0                                                          DMD05305
      NPT=N/2                                                             DMD05306
      IGO=N/8                                                             DMD05307
      DO 11 I=1,12                                                        DMD05308
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   11 S(I)=0.0D0                                                        DMD05309
      IF(NN.LE.1) GO TO 19                                              DMD05310
      OPX=ONE+X                                                         DMD05311
      OPY=ONE+Y                                                         DMD05312
      OMX=ONE-X                                                         DMD05313
      OMY=ONE-Y                                                         DMD05314
      GO TO (14,15,16),IGO                                              DMD05315
C                                                                       DMD05316
C     LINEAR SHAPE FUNCTIONS                                            DMD05317
C                                                                       DMD05318
   14 S(3)=(OMX*OMY)/FOUR                                               DMD05319
      S(4)=(OPX*OMY)/FOUR                                               DMD05320
      S(1)=(OPX*OPY)/FOUR                                               DMD05321
      S(2)=(OMX*OPY)/FOUR                                               DMD05322
      GO TO 17                                                          DMD05323
   15 CONTINUE                                                          DMD05324
C                                                                       DMD05325
C     QUADRATIC SHAPE FUNCTIONS                                         DMD05326
C                                                                       DMD05327
      S(5)=-(OMX*OMY*(X+Y+ONE))/FOUR                                    DMD05328
      S(6)=(OMX*OPX*OMY)/TWO                                            DMD05329
      S(7)=(OPX*OMY*(X-Y-ONE))/FOUR                                     DMD05330
      S(8)=(OPX*OPY*OMY)/TWO                                            DMD05331
      S(1)=(OPX*OPY*(X+Y-ONE))/FOUR                                     DMD05332
      S(2)=(OPX*OMX*OPY)/TWO                                            DMD05333
      S(3)=(OMX*OPY*(-X+Y-ONE))/FOUR                                    DMD05334
      S(4)=(OMX*OPY*OMY)/TWO                                            DMD05335
      GO TO 17                                                          DMD05336
   16 CONTINUE                                                          DMD05337
C                                                                       DMD05338
C     CUBIC SHAPE FUNCTIONS                                             DMD05339
      CONST=(-TEN+NINE*(X**TWO+Y**TWO))/32.0D0                          DMD05340
      S(7)=OMX*OMY*CONST                                                DMD05341
      S(8)=(NINE*OMY*OMX*OPX*(ONE-THR*X))/32.0D0                        DMD05342
      S(9)=(NINE*OMY*OMX*OPX*(ONE+THR*X))/32.0D0                        DMD05343
      S(10)= OPX*OMY*CONST                                              DMD05344
      S(11)=NINE*OPX*OPY*OMY*(ONE-THR*Y)/32.0D0                         DMD05345
      S(12)=NINE*OPX*OPY*OMY*(ONE+THR*Y)/32.0D0                         DMD05346
      S(1)=OPX*OPY*CONST                                                DMD05347
      S(2)=NINE*OPY*OPX*OMX*(ONE+THR*X)/32.0D0                          DMD05348
      S(3)=NINE*OPY*OPX*OMX*(ONE-THR*X)/32.0D0                          DMD05349
      S(4)=OMX*OMY*CONST                                                DMD05350
      S(5)=NINE*OMX*OPY*OMY*(ONE+THR*Y)/32.0D0                          DMD05351
      S(6)=NINE*OMX*OPY*OMY*(ONE-THR*Y)/32.0D0                          DMD05352
   17 CONTINUE                                                          DMD05353
C                                                                       DMD05354
C FORM (2XNPT) MATRIX OF DERIV. OF THE INTERP. FUNCT. WRT XI AND ETA    DMD05355
C                                                                       CAN05360
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   19 GO TO (20,30,40),IGO                                          CAN05380
C                                                                   CAN05390
C    LINEAR FUNCTIONS                                               CAN05400
C                                                                   CAN05410
   20 W1(1,3)=-(ONE-Y)/FOUR                                         CAN05420
      W1(1,4)=-W1(1,3)                                              CAN05430
      W1(1,1)= (ONE+Y)/FOUR                                         CAN05440
      W1(1,2)=-W1(1,1)                                              CAN05450
      W1(2,3)=-(ONE-X)/FOUR                                         CAN05460
      W1(2,4)=-(ONE+X)/FOUR                                         CAN05470
      W1(2,1)=-W1(2,4)                                              CAN05480
      W1(2,2)=-W1(2,3)                                              CAN05490
      GO TO 50                                                      CAN05500
   30 CONTINUE                                                      CAN05510
C                                                                   CAN05520
C   QUADRATIC FUNCTIONS                                             CAN05530
C                                                                   CAN05540
      TXPY=TWO*X+Y                                                  CAN05550
      TXMY=TWO*X-Y                                                  CAN05560
      TYPX=TWO*Y+X                                                  CAN05570
      TYMX=TWO*Y-X                                                  CAN05580
      OMY=ONE-Y                                                     CAN05590
      OPY=ONE+Y                                                     CAN05600
      OPX=ONE+X                                                     CAN05610
      OMX=ONE-X                                                     CAN05620
      W1(1,5)=OMY*TXPY/FOUR                                         CAN05630
      W1(1,6)=-OMY*X                                                CAN05640
      W1(1,7)=OMY*TXMY/FOUR                                         CAN05650
      W1(1,8)=OPY*OMY/TWO                                           CAN05660
      W1(1,1)=OPY*TXPY/FOUR                                         CAN05670
      W1(1,2)=-OPY*X                                                CAN05680
      W1(1,3)= OPY*TXMY/FOUR                                        CAN05690
      W1(1,4)=-OPY*OMY/TWO                                          CAN05700
      W1(2,5)=OMX*TYPX/FOUR                                         CAN05710
      W1(2,6)=-OPX*OMX/TWO                                          CAN05720
      W1(2,7)=OPX*TYMX/FOUR                                         CAN05730
      W1(2,8)=-Y*OPX                                                CAN05740
      W1(2,1)=OPX*TYPX/FOUR                                         CAN05750
      W1(2,2)=OPX*OMX/TWO                                           CAN05760
      W1(2,3)=OMX*TYMX/FOUR                                         CAN05770
      W1(2,4)=-Y*OMX                                                CAN05780
      GO TO 50                                                      CAN05790
   40 CONTINUE                                                      CAN05800
C                                                                   CAN05810
C    CUBIC FUNCTIONS                                                CAN05820
C                                                                   CAN05830
      OPX=ONE+X                                                     CAN05840
      OPY=ONE+Y                                                     CAN05850
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      OMX=ONE-X                                                         CAN05860
      OMY=ONE-Y                                                         CAN05870
      TPTMNX=3.0D0+2.0D0*X-9.0D0*X*X                                    CAN05880
      TMTMNX=3.0D0-2.0D0*X-9.0D0*X*X                                    CAN05890
      TPTMNY=3.0D0+2.0D0*Y-9.0D0*Y*Y                                    CAN05900
      TMTMNY=3.0D0-2.0D0*Y-9.0D0*Y*Y                                    CAN05910
      TYPO=3.0D0*Y+ONE                                                  CAN05920
      TYMO=3.0D0*Y-ONE                                                  CAN05930
      TXPO=3.0D0*X+ONE                                                  CAN05940
      TXMO=3.0D0*X-ONE                                                  CAN05950
      TMTPEX=10.0D0-27.0D0*X*X+18.0D0*X-9.0D0*Y*Y                       CAN05960
      TMTMEX=10.0D0-27.0D0*X*X-18.0D0*X-9.0D0*Y*Y                       CAN05970
      TMNPEY= 10.0D0-9.0D0*X*X+18.0D0*Y-27.0D0*Y*Y                      CAN05980
      TMNMEY= 10.0D0-9.0D0*X*X-18.0D0*Y-27.0D0*Y*Y                      CAN05990
      TTT=32.0D0                                                        CAN06000
      TTN=32.0D0/9.0D0                                                  CAN06010
      W1(1,7)= OMY*TMTPEX/TTT                                           CAN06020
      W1(1,8)=-TPTMNX*OMY/TTN                                           CAN06030
      W1(1,9)=TMTMNX*OMY/TTN                                            CAN06040
      W1(1,10)=-OMY*TMTMEX/TTT                                          CAN06050
      W1(1,11)=-OPY*OMY*TYMO/TTN                                        CAN06060
      W1(1,12)=OPY*OMY*TYPO/TTN                                         CAN06070
      W1(1,1)=-OPY*TMTMEX/TTT                                           CAN06080
      W1(1,2)=TMTMNX*OPY/TTN                                            CAN06090
      W1(1,3)=-TPTMNX*OPY/TTN                                           CAN06100
      W1(1,4) =OPY*TMTPEX/TTT                                           CAN06110
      W1(1,5) =-OPY*OMY*TYPO/TTN                                        CAN06120
      W1(1,6) = OPY*OMY*TYMO/TTN                                        CAN06130
      W1(2,7) = OMX*TMNPEY/TTT                                          CAN06140
      W1(2,8)= OPX*OMX*TXMO/TTN                                         CAN06150
      W1(2,9)=-OPX*OMX*TXPO/TTN                                         CAN06160
      W1(2,10) = OPX*TMNPEY/TTT                                         CAN06170
      W1(2,11)=-TPTMNY*OPX/TTN                                          CAN06180
      W1(2,12)= TMTMNY*OPX/TTN                                          CAN06190
      W1(2,1) =-OPX*TMNMEY/TTT                                          CAN06200
      W1(2,2)=OPX*OMX*TXPO/TTN                                          CAN06210
      W1(2,3)=-OPX*OMX*TXMO/TTN                                         CAN06220
      W1(2,4) =-OMX*TMNMEY/TTT                                          CAN06230
      W1(2,5) =OMX*TMTMNY/TTN                                           CAN06240
      W1(2,6) =-OMX*TPTMNY/TTN                                          CAN06250
   50 CONTINUE                                                          CAN06260
C                                                                       CAN06270
C FORM JACOBIAN OF THE TRANSFORMATION IN AJ(I,J)                        CAN06280
C                                                                       CAN06290
      DO 100 I=1,2                                                      CAN06300
      DO 100 J=1,2                                                      CAN06310
      AJ(I,J)=ZERO                                                      CAN06320
      DO 100 K=1,NPT                                                    CAN06330
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  100 AJ(I,J)=AJ(I,J)+W1(I,K)*COORD(K,J)                                  CAN06340
C                                                                         CAN06350
C CALCULATE DETERMINANT OF THE JACOBIAN                                   CAN06360
C                                                                         CAN06370
      DTJ=AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)                                 CAN06380
C                                                                         CAN06390
C INVERT JACOBIAN IN AJIN                                                 CAN06400
C                                                                         CAN06410
      AJIN(1,1)=AJ(2,2)/DTJ                                               CAN06420
      AJIN(1,2)=-AJ(1,2)/DTJ                                              CAN06430
      AJIN(2,1)=-AJ(2,1)/DTJ                                              CAN06440
      AJIN(2,2)=AJ(1,1)/DTJ                                               CAN06450
C                                                                         CAN06460
C FORM (2XNPT) MATRIX OF DERIVATIVES OF INTERP. FUNCT WRT X AND Y         CAN06470
C                                                                         CAN06480
      DO 200 I=1,2                                                        CAN06490
      DO 200 J=1,NPT                                                      CAN06500
      DNX(I,J)=ZERO                                                       CAN06510
      DO 200 K=1,2                                                        CAN06520
  200 DNX(I,J)=DNX(I,J)+AJIN(I,K)*W1(K,J)                                 CAN06530
C                                                                         CAN06540
C FORM B(I,J) MATRIX                                                      CAN06550
C                                                                         DMD06551
      IF(NN.LE.1) GO TO 250                                               DMD06552
      R=0.0D0                                                             DMD06553
      DO 650 L=1,NPT                                                      DMD06554
      R=R+S(L)*COORD(L,2)                                                 DMD06555
  650 CONTINUE                                                            DMD06556
      DO 230 I=1,NPT                                                      DMD06557
      IOD=2*I-1                                                           DMD06558
      IEV=2*I                                                             DMD06559
      B(1,IOD)=DNX(1,I)                                                   DMD06560
      B(2,IEV)=DNX(2,I)                                                   DMD06561
      B(3,IEV)=S(I)/R                                                     DMD06562
      B(4,IEV)=DNX(1,I)                                                   DMD06563
  230 B(4,IOD)=DNX(2,I)                                                   DMD06564
      GO TO 400                                                           DMD06565
  250 DO 300 I=1,NPT                                                      CAN06570
      IOD=2*I-1                                                           CAN06580
      IEV=2*I                                                             CAN06590
      B(1,IOD)=DNX(1,I)                                                   CAN06600
      B(2,IEV)=DNX(2,I)                                                   CAN06610
      B(3,IEV)=DNX(1,I)                                                   CAN06620
  300 B(3,IOD)=DNX(2,I)                                                   CAN06630
C                                                                         CAN06640
C FORM STIFFNESS BY THE CONGRUENT TRANSFORMATION  BT*D*B                  CAN06650
C                                                                         CAN06660
  400 DO 500 I=1,N                                                        CAN06670
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      DO 500  J=1,4                                                     CAN06680
      B1(I,J)=ZERO                                                      CAN06690
      DO 500  K=1,4                                                     CAN06700
  500 B1(I,J)=B1(I,J)+B (K,I)*ELAST(K,J)                                CAN06710
      DO 600  I=1,N                                                     CAN06720
      DO 600  J=1,N                                                     CAN06730
      AK(I,J)=ZERO                                                      CAN06740
      DO 600  K=1,4                                                     CAN06750
  600 AK(I,J)=AK(I,J)+B1(I,K)*B(K,J)                                    CAN06760
      IF(NN.EQ.2) GO TO 660                                             DMD06765
      R=1.0D0                                                           DMD06770
  660 DO 700  I=1,N                                                     DMD06775
      DO 700  J=I,N                                                     CAN06780
      AK(I,J)=((AK(I,J)+AK(J,I))/2.0D0)*DTJ*R                           CAN06790
  700 AK(J,I)=AK(I,J)                                                   CAN06800
      RETURN                                                            CAN06810
      END                                                               CAN06820
      SUBROUTINE LDLT(N,M)                                              CAN06830
      IMPLICIT REAL*8(A-H,O-Z)                                          CAN06840
      COMMON/SOL/A(434,74),B(434),ABGN                                  CAN06850
C **********************************************************************CAN06860
C   THIS SUBROUTINE DECOMPOSES THE COEFFICIENT MATRIX A* OF THE LINEAR  CAN06870
C   BANDED SYMMETRICAL SYSTEM (A*)(X) = (B), INTO THE PRODUCT (L)(D)(LT)CAN06880
C   THE UPPER BAND ONLY IS USED, AND IT MUST BE STORED IN A RECTANGULAR CAN06890
C   ARRAY (A) WITH ALL ITS DIAGONAL ELEMENTS IN THE FIRST COLUMN        CAN06900
C   MATRIX (A) IS DESTROYED IN THE PROCESS AND THE RESULTS RETURNED IN  CAN06910
C   THE SAME ARRAY. MATRIX (D) IS PUT IN THE FIRST COLUMN AND THE REST  CAN06920
C   OF THE MATRIX CONTAINS THE ELEMENTS OF THE UNIT TRIANGULAR MATRIX (LCAN06930
C   IN BANDED FORM WITHOUT ITS IMPLIED UNIT DIAGONAL ELEMENTS.          CAN06940
C                                                                       CAN06950
C         A  IS THE NAME OF THE COEFFICIENT MATRIX                      CAN06960
C         N  IS THE NUMBER OF EQUATIONS IN THE SYSTEM                   CAN06970
C         M  IS THE HALF BAND WIDTH OF THE SYSTEM                       CAN06980
C                                                                       CAN06990
C   CODED BY GILLES CANTIN, NAVAL POSTGRADUATE SCHOOL, MAY 1972         CAN07000
C **********************************************************************CAN07010
      AN=N                                                              CAN07020
      APZRO=ABGN*1.0D-30/AN                                             CAN07030
      AVSN=APZRO/ABGN                                                   CAN07040
      ZRO=0.0D0                                                         CAN07050
      NM=N-1                                                            CAN07060
      DO 300  I=1,NM                                                    CAN07070
      DIAG=A(I,1)                                                       CAN07080
      IF(DIAG.EQ.ZRO) GO TO 400                                         CAN07090
      IF(DIAG.LE.APZRO) WRITE(6,2000) I                                 CAN07100
      DO 100  J=2,M                                                     CAN07110
      IF(DABS(A(I,J)).LE.AVSN) A(I,J)=0.0D0                             CAN07120
  100 A(I,J)=A(I,J)/DIAG                                                CAN07130
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      DO 300 J=2,M                                                      CAN07140
      L=I+J-1                                                           CAN07150
      IF(L.GT.N) GO TO 300                                              CAN07160
      AA=A(I,J)*DIAG                                                    CAN07170
      IF(AA.EQ.ZRO) GO TO 300                                           CAN07180
      DO 200 K=J,M                                                      CAN07190
      ML=1+K-J                                                          CAN07200
  200 A(L,ML)=A(L,ML)-AA*A(I,K)                                         CAN07210
  300 CONTINUE                                                          CAN07220
      RETURN                                                            CAN07230
  400 WRITE(6,1000) I                                                   CAN07240
      STOP                                                              CAN07250
 1000 FORMAT(//,5X,' MATRIX IS SINGULAR ',1I5)                          CAN07260
 2000 FORMAT(//,5X,' WARNING  MATRIX IS NEARLY SINGULAR  EXECUTION CONTICAN07270
     1NUES ', 1I5,/,5X,' RESIDUAL ITERATION MAY YIELD ERRONEOUS RESULTS CAN07280
     2  '//)                                                            CAN07290
      END                                                               CAN07300
      SUBROUTINE SOLV(N,M)                                              CAN07310
      IMPLICIT REAL*8(A-H,O-Z)                                          CAN07320
      COMMON/SOL/A(434,74),B(434),ABGN                                  CAN07330
C ********************************************************************CAN07340
C   THIS SUBROUTINE PERFORMS A FORWARD SUBSTITUTION, FOLLOWED BY A     CAN07350
C   BACKWARD SUBSTITUTION FOR THE SYSTEM (A*) X= (B), AFTER THE        CAN07360
C   MATRIX (A) HAS BEEN DECOMPOSED BY THE SUBROUTINE LDLDT             CAN07370
C                                                                       CAN07380
C         A  IS THE NAME OF THE DECOMPOSED MATRIX OF COEFFICIENT       CAN07390
C         B  IS THE NAME OF THE COLUMN LOAD VECTOR OR RIGHT HAND MEMBER  CAN07400
C            THE ANSWERS ARE RETURNED IN THIS VECTOR UPON EXIT         CAN07410
C         N  IS THE NUMBER OF EQUATIONS IN THE SYSTEM                  CAN07420
C         M  IS THE HALF BAND WIDTH OF THE SYTEM                       CAN07430
C                                                                       CAN07440
C   AFTER ONE CALL TO LDLT , ONE CALL OF SLV PER LOAD VECTOR IS REQUIREDCAN07450
C                                                                       CAN07460
C   CODED BY GILLES CANTIN, NAVAL POSTGRADUATE SCHOOL, MAY 1972        CAN07470
C ********************************************************************CAN07480
      NM=N-1                                                            CAN07490
      DO 100 I=1,NM                                                     CAN07500
      BI=B(I)                                                           CAN07510
      DO 100 J=2,M                                                      CAN07520
      L=I+J-1                                                           CAN07530
      IF(L.GT.N) GO TO 100                                              CAN07540
      B(L)=B(L)-A(I,J)*BI                                               CAN07550
  100 CONTINUE                                                          CAN07560
      DO 200 I=1,N                                                      CAN07570
  200 B(I)=B(I)/A(I,1)                                                  CAN07580
      DO 300 L=2,N                                                      CAN07590
      IR=N-L+1                                                          CAN07600
      BIRP=B(IR+1)                                                      CAN07610
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 220 CONTINUE                                                                CAN08100
5000 FORMAT(5X,I5,6G12.4)                                                    CAN08110
4000 FORMAT(//,2X,' S-T-R-E-S-S-E-S  /  S-T-R-A-I-N-S  FOR  ELEMENT',I3CAN08120
    1,        //,5X,' JOINT',1X,'SIG X ',6X,'SIG Y ',7X,'TAU ',7X,'EPS XCAN08130
    2',7X,'EPS Y', 7X,'GAMMA ',//)                                           CAN08140
     ICT=0                                                                   CAN08150
     DO 300 I2=1,NPEL                                                        CAN08160
     I2A=NCON(I,I2)                                                          CAN08170
     DO 250 J2=1,4                                                           DMD08180
     ICT = ICT + 1                                                           CAN08190
     SSJNT(I2A,J2)=SSEL(ICT)+SSJNT(I2A,J2)                                   CAN08200
 250 SNJNT(I2A,J2)=SNEL(ICT)+SNJNT(I2A,J2)                                   CAN08210
 300 SNJNT(I2A,7)=SNJNT(I2A,7)+1.                                            CAN08220
     IF(NN.GE.2) GO TO 4500                                                  DMD08225
     DO 500 I3=1,NJT                                                         CAN08230
     SCNT=SNJNT(I3,7)                                                        CAN08240
     DO 400 J3=1,3                                                           CAN08250
     SSJNT(I3,J3)=SSJNT(I3,J3)/SCNT                                          CAN08260
 400 SNJNT(I3,J3)=SNJNT(I3,J3)/SCNT                                          CAN08270
     GAU=SSJNT(I3,3)                                                         CAN08280
     SXPSY=(SSJNT(I3,1)+SSJNT(I3,2))/2.0D0                                   CAN08290
     SXMSY=(SSJNT(I3,1)-SSJNT(I3,2))/2.0D0                                   CAN08300
     AINT=DSQRT(SXMSY*SXMSY+GAU*GAU)                                         CAN08310
     SSJNT(I3,4)=SXPSY+AINT                                                  CAN08320
     SSJNT(I3,5)=SXPSY-AINT                                                  CAN08330
     SSJNT(I3,6)=(SSJNT(I3,4)-SSJNT(I3,5))/2.0D0                             CAN08340
     IF(DABS(SXMSY).LE.0.000001D0) GO TO 420                                 CAN08350
     RAN=-GAU/SXMSY                                                          CAN08360
     IF(DABS(RAN).LE.0.01D0) GO TO 450                                       CAN08370
     RAN=DATAN(RAN)                                                          CAN08380
     GO TO 450                                                               CAN08390
 420 RAN=DSIGN(PI,GAU)/2.0D0                                                 CAN08400
 450 SSJNT(I3,7)=RAN*180.0D0/(2.0D0*PI)                                      CAN08410
     GAU=SNJNT(I3,3)                                                         CAN08420
     SXPSY=(SNJNT(I3,1)+SNJNT(I3,2))/2.0D0                                   CAN08430
     SXMSY=(SNJNT(I3,1)-SNJNT(I3,2))/2.0D0                                   CAN08440
     AINT=DSQRT(SXMSY*SXMSY+GAU*GAU)                                         CAN08450
     SNJNT(I3,4)=SXPSY+AINT                                                  CAN08460
     SNJNT(I3,5)=SXPSY-AINT                                                  CAN08470
     SNJNT(I3,6)=(SNJNT(I3,4)-SNJNT(I3,5))/2.0D0                             CAN08480
 500 SNJNT(I3,7)=SSJNT(I3,7)                                                 CAN08490
     WRITE(6,1000)                                                           CAN08500
     DO 600 I=1,NJT                                                          CAN08510
 600 WRITE(6,2000) I,(SSJNT(I,J),J=1,7)                                      CAN08520
2000 FORMAT(5X,I5,6G12.4,5X,1G12.4)                                          CAN08530
     WRITE(6,3000)                                                           CAN08540
     DO 700 I=1,NJT                                                          CAN08550
 700 WRITE(6,2000) I,(SNJNT(I,J),J=1,6),SSJNT(I,7)                           CAN08560
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1000 FORMAT(///,1X,' A-V-E-R-A-G-E  S-T-R-E-S-S-E-S  AT  THE  JOINTS', CAN08570
    1        //,5X,' JOINT',1X,'SIG X ',6X,'SIG Y ',7X,'TAU ',7X,'S.MAXCAN08580
    2',7X,'S.MIN', 7X,'T.MAX ',12X,'  TET(DEG.)'//)                    CAN08590
3000 FORMAT(///,1X,' A-V-E-R-A-G-E  S-T-R-A-I-N-S  AT  THE  JOINTS',   CAN08600
    1        //,5X,' JOINT',1X,'EPS X ',6X,'EPS Y ',7X,'GAMMA',7X,'E.MAXCAN08610
    2',7X,'E.MIN', 7X,'G.MAX ',12X,'  TET(DEG.)',//)                   DMD08611
     RETURN                                                            DMD08612
4500 DO 4520 I4=1,NJT                                                  DMD08613
     SCNT=SNJNT(I4,7)                                                  DMD08614
     DO 4510 J4=1,4                                                    DMD08615
     SSJNT(I4,J4)=SSJNT(I4,J4)/SCNT                                    DMD08616
4510 SNJNT(I4,J4)=SNJNT(I4,J4)/SCNT                                    DMD08617
4520 CONTINUE                                                          DMD08618
     WRITE(6,4600)                                                     DMD08619
4600 FORMAT(///,1X,'AVERAGE STRESSES AT THE JOINTS',                   DMD08620
    1        //,5X,'JOINT',4X,'SIG Z ',6X,'SIG R ',5X,'SIG THETA',3X,'TDMD08621
    2AU RZ',//)                                                        DMD08622
     DO 6000 I=1,NJT                                                   DMD08623
6000 WRITE(6,7000) I,(SSJNT(I,J),J=1,4)                                DMD08625
7000 FORMAT(5X,I5,4G12.4)                                              DMD08626
     WRITE(6,8000)                                                     DMD08627
8000 FORMAT(///,1X,'AVERAGE STRAINS AT THE JOINTS',//,5X,'JOINT',4X,   DMD08628
    1'EPS Z',6X,'EPS R',7X,'EPS THETA',4X,'GAMMA ZR',//)               DMD08629
     DO 9000 I=1,NJT                                                   DMD08630
9000 WRITE(6,7000) I,(SNJNT(I,J),J=1,4)                                DMD08632
     RETURN                                                            DMD08633
     END                                                               CAN08640
     SUBROUTINE BCOND(NBAND)                                           CAN08650
     IMPLICIT REAL*8(A-H,O-Z)                                          CAN08660
     COMMON/FLPL/COARD(217,2),CLOAD(217,2),ELCON(10,4),TITLE(10)       CAN08670
     COMMON/SOL/BGK(434,74),ALOAD(434),ABGN                            CAN08680
     COMMON/INT/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,14),NBC(217,2),NSTRESCAN08690
    1,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN                                CAN08700
     PI=3.14159265359D0                                                CAN08710
     NEQ=NJT*2                                                         CAN08720
     DO 200 I=1,NPBC                                                   CAN08730
     IJT=NBC(I,1)                                                      CAN08740
     IEQ=IJT*2-1                                                       CAN08750
     KODE=NBC(I,2)                                                     CAN08760
     IDIA=0                                                            CAN08770
     GO TO (110,120,130,140,150,160,170),KODE                          CAN08780
 110 IEQ=IEQ+1                                                         CAN08790
 112 BGK(IEQ,1)=ABGN                                                   CAN08800
     IF(IDIA.EQ.1) GO TO 132                                           CAN08810
     GO TO 200                                                         CAN08820
 120 GO TO 112                                                         CAN08830
 130 IDIA=1                                                            CAN08840
     GO TO 112                                                         CAN08850
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```
 132 IDIA=0                                                                CAN08860
     GO TO 110                                                             CAN08870
 140 DXY=CLOAD(IJT,2)                                                      CAN08880
 142 IEQ=IEQ+1                                                             CAN08890
 143 ALOAD(IEQ)=ALOAD(IEQ)-BGK(IEQ,1)*DXY                                  CAN08900
     BGK(IEQ,1)=ABGN                                                       CAN08910
     DO 146 J=2,NBAND                                                      CAN08920
     IROW=IEQ-J+1                                                          CAN08930
     ICOL=IEQ-1+J                                                          CAN08940
     IF(IROW.LT.1) GO TO 144                                               CAN08950
     ALOAD(IROW)=ALOAD(IROW)-BGK(IROW,J)*DXY                               CAN08960
 144 IF(ICOL.GT.NEQ) GO TO 146                                             CAN08970
     ALOAD(ICOL)=ALOAD(ICOL)-BGK(IEQ,J)*DXY                                CAN08980
 146 CONTINUE                                                              CAN08990
     IF(IDIA.EQ.1) GO TO 162                                               CAN09000
     GO TO 200                                                             CAN09010
 150 DXY=CLOAD(IJT,1)                                                      CAN09020
     GO TO 143                                                             CAN09030
 160 IDIA=1                                                                CAN09040
     DXY=CLOAD(IJT,1)                                                      CAN09050
     GO TO 143                                                             CAN09060
 162 IDIA=0                                                                CAN09070
     GO TO 140                                                             CAN09080
 170 ALPHA=CLOAD(IJT,2)*PI/180.0D0                                         CAN09090
     CAL=DCOS(ALPHA)                                                       CAN09100
     SAL=DSIN(ALPHA)                                                       CAN09110
     IEQP=IEQ+1                                                            CAN09120
     DO 180 J=3,NBAND                                                      CAN09130
     IROW=IEQ-J+2                                                          CAN09140
     JM=J-1                                                                CAN09150
     IF(IROW.LT.1) GO TO 175                                               CAN09160
     A1=BGK(IROW,JM)*CAL+BGK(IROW,J)*SAL                                   CAN09170
     A2=BGK(IROW,J)*CAL-BGK(IROW,JM)*SAL                                   CAN09180
     BGK(IROW,JM)=A1                                                       CAN09190
     BGK(IROW,J )=A2                                                       CAN09200
 175 IF((IEQ+J-1).GT.NEQ) GO TO 180                                        CAN09210
     A3=BGK(IEQ,J)*CAL+BGK(IEQP,JM)*SAL                                    CAN09220
     A4=BGK(IEQP,JM)*CAL-BGK(IEQ,J)*SAL                                    CAN09230
     BGK(IEQ ,J )=A3                                                       CAN09240
     BGK(IEQP,JM)=A4                                                       CAN09250
 180 CONTINUE                                                              CAN09260
     A1=BGK(IEQ,1)*CAL*CAL+BGK(IEQ,2)*CAL*SAL*2.0  +BGK(IEQP,1)*SAL*SALCAN09270
     A2=BGK(IEQ,2)*(CAL*CAL-SAL*SAL)+(BGK(IEQP,1)-BGK(IEQ,1))*SAL*CAL  CAN09280
     BGK(IEQ,1)=A1                                                         CAN09290
     BGK(IEQ,2)=A2                                                         CAN09300
     BGK(IEQP,1)=ABGN                                                      CAN09310
 200 CONTINUE                                                              CAN09320
     RETURN                                                                CAN09330
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      END                                                               CANO9340
      SUBROUTINE DISPL                                                  CANO9350
      IMPLICIT REAL*8(A-H,O-Z)                                          CANO9360
      COMMON/FLPL/COARD(217,2),CLOAD(217,2),ELCON(10,4),TITLE(10)       CANO9370
      COMMON/SOL/BGK(434,74), DISP(434),ABGN                            CANO9380
      COMMON/INT/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,14),NBC(217,2),NSTRESCANO9390
     1,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN                                CANO9400
      COMMON/MDIM/MEL,MJT,MMT,MBD                                       DMDO9405
      DIMENSION REACT(218,2)                                            CANO9410
      EQUIVALENCE (BGK(1,1),REACT(1,1))                                 CANO9420
      PI=3.14159265359D0                                                CANO9430
      DO 200 I=1,NPBC                                                   CANO9440
      IJT=NBC(I,1)                                                      CANO9450
      IEQ=IJT*2-1                                                       CANO9460
      KODE=NBC(I,2)                                                     CANO9470
       IDIA=0                                                           CANO9480
      DPBC=0.0D0                                                        CANO9490
      GO TO (110,120,130,140,150,160,170),KODE                          CANO9500
  110 IEQ=IEQ+1                                                         CANO9510
      JR=2                                                              CANO9520
  112 REACT(I,JR)=-DISP(IEQ)*ABGN                                       CANO9530
      DISP(IEQ)=DPBC                                                    CANO9540
      IF(IDIA.EQ.1) GO TO 132                                           CANO9550
      IF(IDIA.EQ.2) GO TO 162                                           CANO9560
      GO TO 200                                                         CANO9570
  120 JR=1                                                              CANO9580
      GO TO 112                                                         CANO9590
  130 IDIA=1                                                            CANO9600
      JR=1                                                              CANO9610
      GO TO 112                                                         CANO9620
  132 IDIA=0                                                            CANO9630
      GO TO 110                                                         CANO9640
  140 DPBC=CLOAD(IJT,2)                                                 CANO9650
      GO TO 110                                                         CANO9660
  150 JR=1                                                              CANO9670
      DPBC=CLOAD(IJT,1)                                                 CANO9680
      GO TO 112                                                         CANO9690
  160 IDIA=2                                                            CANO9700
      JR=1                                                              CANO9710
      DPBC=CLOAD(IJT,1)                                                 CANO9720
      GO TO 112                                                         CANO9730
  162 IDIA=0                                                            CANO9740
      DPBC=CLOAD(IJT,2)                                                 CANO9750
      GO TO 110                                                         CANO9760
  170 ALPHA=CLOAD(IJT,2)*PI/180.0D0                                     CANO9770
      CAL=DCOS(ALPHA)                                                   CANO9780
      SAL=DSIN(ALPHA)                                                   CANO9790
      A1=DISP(IEQ)*CAL                                                  CANO9800
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      A2=DISP(IEQ)*SAL                                                  CAN09810
      IEQP=IEQ+1                                                        CAN09820
      A3=-DISP(IEQP)*SAL*ABGN                                           CAN09830
      A4= DISP(IEQP)*CAL*ABGN                                           CAN09840
      DISP(IEQ)=A1                                                      CAN09850
      DISP(IEQP)=A2                                                     CAN09860
      REACT(I   ,1)=-A3                                                 CAN09870
      REACT(I   ,2)=-A4                                                 CAN09880
  200 CONTINUE                                                          CAN09890
      MJP=MJT+1                                                         DMD09900
      REACT(MJP,1)=0.0D0                                                DMD09905
      REACT(MJP,2)=0.0D0                                                DMD09910
      DO 300 I=1,NPBC                                                   CAN09920
      REACT(MJP,1)=REACT(MJP,1)+REACT(I,1)                              DMD09930
  300 REACT(MJP,2)=REACT(MJP,2)+REACT(I,2)                              DMD09940
      RETURN                                                            CAN09950
      END                                                               CAN09960
```

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